

The role of workfare in striking a balance between incentives and insurance in the labour market*

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Abstract

Workfare policies are often introduced in labour market policies to improve the trade-off between incentives and insurance as an alternative to benefit reductions. We consider the role of workfare policies in a general equilibrium search framework and find that three important effects are at stake, namely, a locking-in effect, a threat-effect and a wage effect. Most of the empirical literature has focused on the direct effect of workfare on those activated, but it is shown that this may be a very poor indicator on the overall effects of workfare policies. The reason is that the direct search effects may be dominated by the threat and wage effect arising from the fact that workfare policies not only affects those in the programme but also unemployed and employed, since they face a risk of unemployment and hence transfer to workfare programmes. Workfare policies are shown to reduce both open and total unemployment for given benefit levels, i.e. unemployment benefits are unchanged, but the incentives structure is improved.

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1 Introduction

It is a key policy challenge to ensure a proper balance between incentives and insurance in the labour market. The incentive part is related to active job search including relocation across jobs, skills and geographical dimensions to make the labour market flexible and able to cope with shocks and structural changes ensuring a low and stable unemployment rate, and the insurance part is related to the risk associated with loss of jobs and incomes carried by individuals. The challenge is to ensure both incentives and insurance so as to combine economy wide demands to the labour market with individual desires for security.

The discussion of how to strike a balance between incentives and insurance in the labour market has recently been much discussed under the heading of flexicurity which alludes to the possibility of having flexible rules for hiring and firing of workers in combination with a generous unemployment insurance scheme coping with individual risks associated with labour market flexibility. Denmark is often portrayed as a model example of a flexicurity system with very flexible firing rules and a generous unemployment insurance system. Since unemployment in Denmark is relatively low this has been taken to illustrate the possibility of striking a balance between incentives and insurance. However, in accounting for the Danish experience it is important also to take into account the role of active labour market policies in the form of workfare elements. Both rather flexible dismissal rules and the unemployment insurance (and social assistance) system have been virtually unchanged for many years, and were also in place during the period from the mid 1970s to the early 1990s with double digit unemployment rates. The major changes in Denmark in recent years are a sequence of reforms¹ during the 1990s and continued in recent years which has strengthened the third pillar of Danish labour market policy, namely, the active labour market policy where workfare elements have come to play an important role (for details see e.g. Andersen and Svarer (2007)).

The role of workfare in striking a balance between incentives and insurance is thus an important policy issue. The emphasis on active labour market policies is, however, not a new phenomena and various elements can be found in the labour market policy for many countries. The experience with active labour market policies is rather mixed. A large number of empirical analysis of various elements of active labour market policies exists, but they leave a rather unclear message. In a recent survey Kluge (2006) concludes that there are some positive effects of private job creation programmes and measures aiming at enhancing job search efficiency, whereas training programmes and public sector employment are less efficient instruments which may even have a negative effect on employment. However, most of these analysis are partial in nature focussing

¹The main orientation of the reform is a shift from a passive orientation of the system focusing on income maintenance to an active focus on ensuring employment. Important policy changes include i) a strengthening of workfare element in both the unemployment benefit and social assistance scheme, ii) eliminating participation in activation measures as a possibility to renew eligibility for unemployment benefits, iii) shortening of the effective duration of benefit. In short this can be interpreted as strengthening the incentive side while maintain a reasonable level of insurance (without reducing benefits), see Andersen and Svarer (2007).

on the direct effects of various policy measures neither taking into account the general equilibrium effects (wage responses, search incentives for other groups, employment creation etc.) nor the financing of active labour market policies. The empirical literature thus brings up different channels through which workfare can influence labour market policies. There is thus a need to clarify which elements of active labour market policies that can be used to mitigate the trade-off between insurance and incentives.

The contribution of this paper is to consider how workfare as an element of an unemployment benefit scheme can be an instrument in striking a balance between considerations for incentives and insurance. Specifically whether introduction of workfare policies can improve labour market performance for given benefits. We present a general equilibrium analysis of workfare in a basic search framework. A tax financed unemployment insurance scheme deals with the insurance aspect, but may for well-known reasons create incentive problems inducing insufficient job search on the part of the unemployed. Introduction of workfare as a condition for being eligible for benefits may change incentives. We model workfare as a possible time demand on the unemployed to remain eligible for benefits. Since workfare requires time effort on behalf of the unemployed a consequence may be lower job search for those on workfare (*the locking-in effect*). However, incentives also change for the unemployed not yet in workfare since being in a workfare programme is less attractive than passive claiming benefits. Search effort for the unemployed may therefore increase (*the threat or motivation effect*). Hence, the effect of workfare on overall job search depends on the balance between the locking-in and the threat effect² It is an implication that finding some locking-in effect of workfare is not a sufficient condition that workfare lowers overall job search activity. Moreover, and less obvious, even the direct effect of workfare on total search effort is not pivotal, since there is also a *wage effect*. This wage effect arises because workfare affects the outside option of employed workers, that is, the gain from having a job increases and this tends to lead to wage moderation. The empirical literature has mainly focussed on the direct effect of workfare on the activated and the unemployed, but the wage effect arising via a change in the outside option of employed workers may be equally important.

Notice also that workfare policies have two dimensions, namely, the intensity of workfare among unemployed and the work requirement. The two are not in general equivalent and therefore the composition of workfare policies are of importance for search effort and wage determination, and thus unemployment. It is an implication that changes in benefits and workfare are not equivalent even from a utility perspective in a search environment, and therefore this policy tool may critically affect the incentive and insurance trade-off.

We consider these mechanisms both analytically and in numerical simulations. The key questions addressed are the effects of workfare requirements on

²There may also be human capital effects. One is that human capital depreciates with the length of the unemployment spell. Another is that the activation programme may enhance human capital and therefore subsequent job prospects. Since all workers and jobs are alike in the present setting these human capital aspects are disregarded.

search effort undertaken by different groups in the labour market, and its effect on total search effort and wage determination and therefore on the overall unemployment rate (open and total). We consider the distributional consequences in terms of the fraction of workers in different states (employment, benefits, and workfare) and their income and utility levels.

This paper contributes to the theoretical literature on workfare policies. Besley and Coate (1992, 1995) pointed out that workfare can be used as a screening device and therefore allow a better targeting of income transfers. This effect of workfare policies has been further analysed by. In a labour context Kreiner and Tranæs (2005) analyse workfare as a screening device affecting the optimal unemployment insurance offered for a given unemployment risk. Fredrikson and Holmlund (2006) compare workfare policies with time limits and sanctions in an unemployment insurance scheme and argue that workfare does not improve search incentives but may hamper them by being time consuming (locking in effect). Hence they argue that workfare is dominated by time limits and monitoring of search. However, Holzner, Meier and Werding (2006) find in an efficiency wage model that workfare may lessen the non-shirking condition since unemployment benefits become less attractive, and this in turn shifts the wage curve.

An important question from a normative perspective is whether there is any rationale for introducing workfare policies or whether this policy is dominated by other policy measures like benefit cuts. In the literature on income redistribution programmes it is a general finding that workfare as such does not leave more leverage in balancing incentives and redistribution (insurance), since a change in benefits or work requirements would both work via changing the utility offered people receiving transfers. Under a standard utility metric there is not, therefore, much role for workfare as part of optimal policies³⁴. The situation changes if the policy objective is cast in terms of income or consumption possibilities (income maintenance) rather than utility, since workfare in this case can be used to strengthen incentives for given benefit or income levels, see Besley and Coate (1992, 1995). It is reasonable to argue that distributional discussions usually focus on income, and therefore workfare may create an extra degree of freedom in redistribution policies. Moreover income is interpersonally comparable which utility is not. An alternative justification may be given in terms of desert-sensitive altruism (see e.g. Luttens and Valfort (2007)) where "hard working" individuals will oppose redistribution from the "hard-working" to the "lazy", i.e. the political support for generous unemployment schemes would be higher if it is associated with workfare elements. This is also related to work norms often permeating policy discussions. At any rate the primary objective of the present paper is to present a positive analysis of how workfare policies affect labour market performance, in particular open and total unemployment.

The paper is organized as follows: Section 2 sets up the basic model and

³It has been shown that this equivalence result needs not hold if workfare activities are productive (Chambers (1989) and Betts (1998)).

⁴Kreiner and Tranæs (2005) show in an unemployment insurance context with an adverse selection problem that it is Pareto-improving to introduce workfare.

considers the three effects of workfare, i.e. the locking-in effect, the threat effect, and the wage effect. The overall effects of the two dimensions of workfare (intensity and work requirement) are worked out in section 3. Finally, section 4 offers a few concluding remarks and discusses the empirical evidence on the mechanisms analysed in the paper as well as possible extensions of the framework.

2 A search model with workfare

This section develops a very stylized search model⁵ to bring forth some basic effects of workfare as an instrument in labour market policies. Agents are homogeneous but differ in their labour market status, and frictions are associated with transition between labour market states.

2.1 Workers

Consider a labour market regime in which unemployed are entitled to a benefit b when unemployed. Unemployed persons may be required to participate in activation programmes to remain eligible for the benefit. The activation requirement may either be imposed after a certain period of time or at the discretion of the labour market authorities. As argued by Frederiksson and Holmlund(2001b) a fixed time duration can be approximated by a system in which there is a stochastic transition into the scheme. A scheme where the activation (duration and type of activity) is decided at the discretion of the authorities would thus seen from an individual perspective be a stochastic workfare scheme. The probability that an unemployed is required to participate in activation with a work requirement l_a is denoted p_{au} ($0 \leq p_{au} \leq 1$) and this is also the fraction of unemployed in activation. These two dimensions of workfare (l_a, p_{au}) are exogenous policy instruments.

Agents search for jobs with intensity s_u when unemployed and s_a when on activation. The going wage rate after tax for employed is \tilde{w} ($\equiv w(1 - \tau)$) and the work requirement l_e (exogenous). The instantaneous utility depends on consumption (= disposable income) and leisure ($F_i = 1 - l_i - s_i$, where the time endowment has been normalized to unity, l denotes time work, and s time spend search for jobs), i.e.

$$\begin{array}{lll} h(w[1 - \tau], 1 - l_e) & \text{if} & \text{employed} \\ g(b, 1 - s_u) & \text{if} & \text{receiving unemployment benefits} \\ g(b, 1 - s_a - l_a) & \text{if} & \text{in activation programme} \end{array}$$

where both h and g are increasing and concave functions in their arguments. We allow the utility functions to differ between employed and unemployed workers to capture eventual stigmatization effects of being without a regular job⁶.

⁵The model structure is closely related to Frederiksson and Holmlund (2005).

⁶None of the analytical results depend on this assumption.

Assuming a constant interest rate r it follows that the value functions (in Steady State) associated with the three labour market states are

$$rV^E = h(w[1 - \tau], 1 - l_e) + p_{ue} [V^U - V^E] \quad (1)$$

$$rV^U = g(b, 1 - s_u) + \alpha s_u [V^E - V^U] + p_{au} [V^A - V^U] \quad (2)$$

$$rV^A = g(b, 1 - s_a - l_a) + \alpha s_a [V^E - V^A] \quad (3)$$

where job offers arrive with probability αs_u for unemployed and αs_a for workers in activation programmes. α is the job arrival rate conditional on search and it is endogenous, see below. Note that the employment probabilities are the same for the two groups provided they exert the same search effort. Hence, there are no human capital differences between the two groups nor any change in human capital from participating in activation⁷. There is an exogenous job separation rate p_{ue} ($0 < p_{ue} < 1$).

From (1), (2) and (3) we have that the value functions for the three labour market states can be written as

$$\begin{aligned} [r + p_{ue}]V^E &= h(w[1 - \tau], 1 - l_e) + p_{ue}V^U \\ [r + \alpha s_u + p_{au}]V^U &= g(b, 1 - s_u) + \alpha s_u V^E + p_{au}V^A \\ [r + \alpha s_a]V^A &= g(b, 1 - s_a - l_a) + \alpha s_a V^E \end{aligned}$$

To see the role of activation it is useful to note that the pay-off as unemployed can be written

$$V^U = \frac{r + \alpha s_u}{r + \alpha s_u + p_{au}} \widehat{V}^U + \frac{p_{au}}{r + \alpha s_u + p_{au}} V^A < \widehat{V}^U \quad (4)$$

where \widehat{V}^U is the pay-off to unemployed in the absence of activation ($p_{au} = 0$) given as

$$\widehat{V}^U = \frac{g(b, 1 - s_u) + \alpha s_u V^E}{(r + \alpha s_u)}$$

The pay-off as unemployed (4) is thus a convex combination of the pay-offs in the absence of activation and under activation. Hence, workfare can be interpreted as a randomized sanction in the unemployment insurance scheme in the sense that with probability p_{au} the individual is required to participate in activation to remain eligible for benefits.

The participation constraint is that employed are always better off than the unemployed

$$V^E - V^U > 0$$

and that the pay-off for those on activation is non-negative

$$V^A \geq 0$$

⁷These human capital effects may be either positive via forms of training or maintenance of human and social capital, or negative in terms of duration dependent depreciation of these.

Note that it is implied that the unemployed are always better off than individuals in activation programmes

$$V^U - V^A > 0 \quad \text{for } l_a > 0, 0 < p_{au}.$$

It is obvious that activation requirements ($l_a > 0$) worsens the situation for those on activation. However, and this is crucial it also affects the position as unemployed since there is possible transition into activation (the threat effect), cf (5) below.

A change in the transition rate from unemployment into activation does not affect those on activation directly, but has a direct effect on the unemployed in terms of increasing the likelihood of changing status from being unemployed to being on activation, cf (5).

$$\begin{aligned} \frac{\partial V^A}{\partial l_a} &= -g'_F(b, 1 - s_a + l_a) < 0 & \frac{\partial V^A}{\partial p_{au}} &= 0 \\ \frac{\partial V^U}{\partial l_a} &= \frac{p_{au}}{r + \alpha s_u + p_{au}} \frac{\partial V^A}{\partial l_a} < 0 & \frac{\partial V^U}{\partial p_{au}} &= \frac{r + \alpha s_u}{[r + \alpha s_u + p_{au}]^2} [V^A - \widehat{V}^U] < 0 \end{aligned} \quad (5)$$

A key question for policy design is whether workfare elements can release any incentive effects different from a benefit reduction, cf. the introduction. Although there from a utility perspective is an equivalence between benefit reductions and workfare elements the effects would differ across the three groups in the labour market: employed, unemployed and activated. This is so for two reasons. First, activation works as a time dependence or stochastic sanction in the pay-off to unemployed since the pay-off is lowered upon transition to workfare. Considering the instantaneous utility for those on activation it is determined by $(g(b, 1 - s_a + l_a))$ and therefore to attain a given change in pay-off for those on workfare a work requirement has an equivalent benefit reduction. However, for those not yet on activation - the unemployed - matters are different. Whereas a reduction in benefit would affect unemployed and activated in similar ways, it is the case that workfare has no direct effect on the instantaneous utility for the unemployed ($g(b, 1 - s_u)$) but a prospective effect via the risk of ending up in activation (the threat or motivation effect). To put it differently, the point is that once activation is in place, changes in workfare demands (l_a) and benefits (b) would not have similar effects for the two groups. Secondly, benefit changes and workfare requirements affect search incentives differently. The reason is that workfare requirement affects the marginal cost of search directly, whereas benefits have an effect via an income effect (see below).

A further issue is the role of the two dimensions of workfare, namely, the incidence of workfare (p_{au}) and the activity requirement (l_a) and whether they affect incentives in similar ways. The answer has two parts, and the first is already given in (5) showing that the two elements of workfare affect pay-off through different mechanisms. Moreover there is an important difference in how they affect unemployed and those on activation. To see the difference between the two elements of workfare consider the marginal rate of substitution of the two instruments for given utility gains ($V^E - V^U$ and $V^E - V^A$) and search effort (s_u and s_a). As shown in the appendix *B* we have that

$$\begin{aligned} \frac{dp_{au}}{dl_a} \Big|_U &= -\frac{p_{au}}{r + \alpha s_a} \frac{g'_F(b, 1 - s_a - l_a)}{V^U - V^A} < 0 \\ \frac{dp_{au}}{dl_a} \Big|_A &= \frac{r + p_{ue} + p_{au} + \alpha s_u}{p_{ue}} \frac{g'_F(b, 1 - s_a - l_a)}{V^U - V^A} > 0 \end{aligned}$$

which gives the marginal rate of substitution between the transition rate and the workfare requirement for the unemployed and those on workfare, respectively. The intuition for the negative rate of substitution for unemployed is straightforward, increasing the work requirement makes the state of unemployment less attractive due to the possibility of being transferred to activation, and this can be compensated by a lower incidence of activation. Therefore for the unemployed, the two instruments are substitutes. For those on activation the situation is different. A higher work requirement would affect pay off negatively, and for the utility differences to be unchanged the state of employment has to be less attractive which is the case (due to the risk of job loss) if unemployment is more likely to lead to activation, i.e. p_{au} is higher. Hence, for those on workfare the two instruments are complements. It is also seen that for both types the marginal rate of substitution depends both on the incidence of workfare (p_{au}) and the work requirement (l_a) suggesting that there may be non-linearities in the effects of the two dimensions.

2.2 Search effort

Individuals choose search effort taking all macro variables (w, τ, α) as given and hence the search effort is determined by⁸

$$g'_F(b, 1 - s_u) = \alpha [V^E - V^U] \quad (6)$$

$$g'_F(b, 1 - s_a - l_a) = \alpha [V^E - V^A] \quad (7)$$

The LHS gives the marginal costs of search and the RHS the marginal gain as the product of the job finding probability α and the utility gain from shifting from the current state into employment.

Since $V^U - V^A > 0$ it follows that

$$g'_F(b, 1 - s_a - l_a) > g'_F(b, 1 - s_u)$$

and therefore

$$s_u < s_a + l_a$$

i.e. those on activation spend more time in total (activation plus search) than the unemployed (search only), but it is in general ambiguous whether search activity is highest for the unemployed or those on activation ($s_a \lesseqgtr s_u$).

An important question is whether workers in activation would search less than other unemployed workers. This is the so-called locking-in effect. It follows from (7) that no unambiguous statements can be made since there are two

⁸The second order conditions are fulfilled given the concavity of the v -function.

effects. First, activation is time consuming and this tends to increase the marginal costs of search and therefore to lower search effort. Second, activation requirements make activation less attractive than unemployment ($V^U - V^A > 0$), and therefore workers on activation has more to gain by becoming employed which tend to make them search more. Hence, in general it is in net-terms ambiguous whether there is a locking-in effect. To see this consider how a change in the work requirement in activation programmes affects search effort for both groups, since (see appendix C for proof of signs)

$$\frac{\partial s_u}{\partial l_a} = \frac{-1}{v_F''(b, 1 - s_u)} \frac{\partial \alpha [V^E - V^U]}{\partial l_a} > 0 \quad (8)$$

$$\frac{\partial s_a}{\partial l_a} = \frac{-1}{v_F''(1 - s_a + l_a)} \frac{\partial \alpha [V^E - V^A]}{\partial l_a} - 1 \lesseqgtr 0 \quad (9)$$

Strengthening the work requirement thus induces the unemployed to exert more search effort since it increases the marginal gain from becoming employed ($\frac{\partial \alpha [V^E - V^U]}{\partial l_a} > 0$). A similar effect is present for those on activation but it is counteracted by the extra time spend in activation. Hence, it is possible that strengthened activation requirements may increase the search effort of unemployed - a threat effect - while decreasing the search effort of those on activation - a locking-in effect.

For the incidence or risk of being on workfare (p_{ue}) we also find a difference in how it affects the unemployed and those on activation, where we have (for proof of signs see appendix C)

$$\frac{\partial s_u}{\partial p_{au}} = \frac{-1}{v_F''(b, 1 - s_u)} \frac{\partial \alpha [V^E - V^U]}{\partial p_{au}} > 0 \quad (10)$$

$$\frac{\partial s_a}{\partial p_{au}} = \frac{-1}{v_F''(b, 1 - s_a + l_a)} \frac{\partial \alpha [V^E - V^A]}{\partial p_{au}} < 0 \quad (11)$$

i.e. a large risk of being in activation induces the unemployed to search more for jobs, since the alternative is now less attractive. Oppositely, the search effort of those already in activation decreases since getting a job become less attractive (due to the risk of losing it again and ending up on activation).

2.3 Matching

Hiring and transitions into employment are determined via a matching mechanism given as

$$m(s, v)$$

where s denotes effective search and v the vacant jobs (see below)⁹. The matching function is assumed to be increasing in both arguments and to display constant returns. Effective or total search is determined by

⁹Expressed in per capita terms, i.e the population is $N = E + U + A$, and $e = E/N$, $u = U/N$, and $a = A/N$.

$$s = s_u u + s_a a$$

where u is the fraction of the population being on unemployment benefits, and a is accordingly the fraction on activation. The job finding rate is

$$\alpha = \frac{m(s, v)}{s} = m(1, \theta)$$

where $\theta = \frac{v}{s}$ and hence $\alpha(\theta)$, $\alpha'(\theta) > 0$. Firms fill vacancies at the rate $q = \frac{m(s, v)}{v} = m(\theta^{-1}, 1)$, $q'(\theta) < 0$.

Inflow and outflow into jobs balance in equilibrium, i.e.

$$[1 - u - a] p_{ue} = \alpha [s_u u + s_a a] \quad (12)$$

as they also do for activation, i.e.

$$\alpha s_a a = p_{au} u \quad (13)$$

2.4 Firms and vacancies

An employed worker produces an output y , while the cost of having an unfilled vacancy is ky ($k > 0$). The value functions are

$$\begin{aligned} rJ^V &= -ky + q(J^E - J^V) \\ rJ^E &= y - w + p_{ue}(J^V - J^E) \end{aligned}$$

Vacancies are created to the point where (free entry) $J^V = 0$ implying the following relationship between the wage rate (w) and labour market tightness (θ)

$$w = \left[1 - (r + p_{ue}) \frac{k}{q} \right] y$$

This gives a relation implying that the higher the wage rate (w), the higher the rate at which firms are filling jobs ($q(\theta)$), i.e. a high wage is associated with a low θ and thus less job creation (fewer vacancies relative to total search effort). Note for later reference that this implies that the job finding rate α is decreasing in the wage rate. The value of a filled job is

$$J^E = \frac{ky}{q} \quad (14)$$

2.5 Wage determination

The wage rate is assumed to be set in a Nash-bargain between workers and the firm, i.e.

$$w = \arg \max [V^E - V^U]^\beta [J^E - J^V]^{1-\beta}$$

where β is the (exogenous) bargaining power and V^U is taken as given. The first order condition reads

$$\beta \frac{\frac{\partial V^E}{\partial w}}{V^E - V^U} + (1 - \beta) \frac{\frac{\partial J^E}{\partial w}}{J^E} = 0$$

where it has been used that $J^V = 0$. The first order condition can be written

$$\Psi(w, \tau, V^E - V^U, q) \equiv \beta \frac{h_w(w[1 - \tau], 1 - l_e)}{V^E - V^U} + (1 - \beta) \frac{-1}{J^E} = 0 \quad (15)$$

and the second-order condition is

$$\Psi_w(w, \tau, V^E - V^U, q) < 0$$

Workfare releases a wage effect. To see this note that

$$\begin{aligned} \Psi_{p_{au}}(w, \tau, V^E - V^U, q) &= -\beta \frac{h_w(w[1 - \tau], 1 - l_e)}{[V^E - V^U]^2} \frac{\partial (V^E - V^U)}{\partial p_{au}} < 0 \\ \Psi_{l_a}(w, \tau, V^E - V^U, q) &\equiv -\beta \frac{h_w(w[1 - \tau], 1 - l_e)}{[V^E - V^U]^2} \frac{\partial (V^E - V^U)}{\partial l_a} < 0 \end{aligned}$$

Using this and the second-order condition it follows that

$$\frac{\partial w}{\partial p_{au}} < 0, \frac{\partial w}{\partial l_a} < 0 \quad (16)$$

i.e. an increase in both the intensity and work requirement of workfare tends to lower the wage rate. This could also be phrased that both of these changes worsen the outside option of the employed in wage negotiations and therefore reduces the wage rate. A lower wage tends to make firm create more vacancies which in turn improves matches etc.. The wage effect of workfare may thus be important on par with the direct search effect.

2.6 Public sector

The policy instruments of the government are the benefit level (b), the incidence of workfare (p_{au}), the work requirement (l_a) and the tax rate τ . The budget constraint for the public sector is

$$\tau w(1 - u - a) = bu + (b + c)a + r$$

where c is the cost of activation programmes, and r other expenditure requirements of the government. We take the tax rate to be given and therefore the expenditure level r is endogenous.¹⁰

It is shown in Appendix *D* that the model has a well-defined equilibrium, and conditions ensuring a unique equilibrium are given.

¹⁰This assumption is made in the theoretical analysis to eliminate a non-linearity which would arise if r is taken to be exogenous and τ endogenous.

3 Workfare policies and labour market policies

Our main interest is to explore how workfare policies can affect labour market performance, in particular open (u) and total ($u + a$) unemployment. The effects of the two dimensions of workfare ($z = l_a, p_{au}$) on unemployment (u), activation (a) and total unemployment ($u + a$) are given as (see Appendix E).

$$\begin{aligned} \frac{\partial u}{\partial z} &= \frac{-\frac{p_{ue} + \alpha s_a}{\alpha s_a} u \frac{\partial p_{au}}{\partial z} - u \frac{\partial \alpha s_u}{\partial z} + a \frac{p_{ue}}{\alpha s_a} \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}} \\ \frac{\partial a}{\partial z} &= \frac{1}{\alpha s_a} \left[\frac{(p_{ue} + \alpha s_u) u \frac{\partial p_{au}}{\partial z} - u p_{au} \frac{\partial \alpha s_u}{\partial z} - [p_{ue} + \alpha s_u + p_{au}] a \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}} \right] \\ \frac{\partial(u+a)}{\partial z} &= \frac{\left[\frac{\alpha(s_u - s_a)}{\alpha s_a} \right] u \frac{\partial p_{au}}{\partial z} - \left[1 + \frac{p_{au}}{\alpha s_a} \right] u \frac{\partial \alpha s_u}{\partial z} - \left[\frac{\alpha s_u + p_{au}}{\alpha s_a} \right] a \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}} \end{aligned}$$

Consider the terms on the RHS of these expressions we find that the first effect arises if the incidence of workfare is affected ($\frac{\partial p_{au}}{\partial z}$). If $\frac{\partial p_{au}}{\partial z} > 0$ the transition from passive to active benefits is increased which tends to lower open unemployment ($\frac{\partial u}{\partial z} < 0$) but increases activation ($\frac{\partial a}{\partial z} > 0$), and hence the net-effect on total unemployment is ambiguous ($\frac{\partial(u+a)}{\partial z} \lesseqgtr 0$) depending on whether search activity is highest for unemployed or activated. This is the effect often discussed when the question is raised whether activation disguises the true unemployment problem. If $\frac{\partial \alpha s_u}{\partial z} > 0$ the effective job finding rate for unemployed increases, which contributes to lower unemployment ($\frac{\partial u}{\partial z} < 0$) and activation ($\frac{\partial a}{\partial z} < 0$) since fewer will be transferred to activation, and this effect thus unambiguously work to lower total unemployment ($\frac{\partial(u+a)}{\partial z} < 0$). If the effective job-finding rate for those in activation increases $\frac{\partial \alpha s_a}{\partial z} > 0$ it will tend to increase open unemployment (total employment goes up and there will be more job separations) ($\frac{\partial u}{\partial z} > 0$) but activation falls ($\frac{\partial a}{\partial z} < 0$) and this effect is dominating such that total unemployment unambiguously falls ($\frac{\partial(u+a)}{\partial z} \lesseqgtr 0$). The above suggests that the effects of workfare policies on labour market performance may be non-monotone.

The findings reported here indicate both that the effects of workfare policies on open unemployment, activation and total unemployment are complicated and also that the net-effect depends on the balance between counteracting effects. Moreover, it brings out that the effective job finding rates are the key transmissions mechanisms. We have that it can be decomposed into a wage and a search effect, since

$$\frac{\partial \alpha s_i}{\partial z} = \frac{\partial \alpha}{\partial w} \frac{\partial w}{\partial z} s_i + \alpha(w) \frac{\partial s_i}{\partial z} \quad \text{for } i = u, a, z = p_{au}, l_a \quad (17)$$

Hence, we have that the wage effect ($\frac{\partial \alpha}{\partial w} \frac{\partial w}{\partial z} > 0$ since $\frac{\partial \alpha}{\partial w} < 0$ and $\frac{\partial w}{\partial z} < 0$) unambiguously increases the effective job finding rate for both unemployed and

activated, whereas the direct search effect is more complicated and depends on the dimensions of workfare considered. The work requirement increases search for unemployed but has an ambiguous effect for the activated (see (8) and (9)) while increasing the propensity of activation leads to more search for the unemployed and less for the activated (see (10) and (11)). This also brings out why a focus on the direct search effect of workfare policies may miss an important element of why workfare policies affect labour market performance, namely, the wage effect.

The many counteracting search effects of a marginal change in either of the two dimensions of workfare policies blurs the fact that introduction of workfare elements in an unemployment insurance scheme may contribute to lower both open and total unemployment. That is, if the unemployment insurance scheme does not have workfare elements there is an argument to introduce them if the aim is to lower unemployment under a distributional constraint of given benefits. To see this we neutralize the unambiguous wage effect (assuming a constant wage) and considering search effects only. It can be shown (see appendix E) that increasing the incidence of workfare (p_{au}) leads to a fall in unemployment, i.e. $\frac{\partial u}{\partial p_{au}}|_{p_{au}=0} < 0$ and lowers total unemployment $\frac{\partial(u+a)}{\partial p_{au}}|_{p_{au}=0} < 0$ provided that the workfare requirement is not too large i.e. $l_a < \bar{l}_a$. Similarly increasing the activity requirement from an initial level of zero leads to a decrease in unemployment ($\frac{\partial u}{\partial l_a}|_{l_a=0} < 0$), an increase in the number of unemployed on workfare ($\frac{\partial a}{\partial l_a}|_{l_a=0} > 0$) but an overall decrease in the fraction of non-employed ($\frac{\partial(u+a)}{\partial l_a}|_{l_a=0} < 0$) provided that the incidence of workfare is not too larger, i.e. $p_{au} < \bar{p}_{au}$. Note that these results indicate that the overall effects of changes in the elements of workfare depend critically on the total policy package, that is, the incidence (p_{au}) and the work requirement (l_a).

Finally, it may be questioned whether introduction of workfare policies is tantamount to a two-tier benefit scheme where there is a transition from a high (b) to a low ($b_L < b$) benefit level since this will also induce an incentive effect to search more actively for jobs. It is relatively straightforward to show that there is not an equivalence between a workfare policy and a two-tier benefit scheme¹¹. The intuition is that the two schemes will affect utility and search incentives differently.

¹¹For this to be the case, the equilibrium attained for a given workfare policy (b, p_{au}, l_a) should be replicated for a scheme where there is a transition to a lower benefit level (b, p_{au}, b_L). For this to be the case, there are two conditions, namely that the utility levels should be the same under the two policies, i.e.

$$g(b, 1 - s_a - l_a) = g(b_L, 1 - s_a)$$

and the search effort should be the same, requiring

$$v'_F(b, 1 - s_a - l_a) = v'_F(b_L, 1 - s_a)$$

Clearly there is in general no level of b_L satisfying both conditions.

4 Numerical illustrations

In this section, we provide numerical illustrations of some of the main effects of variations in the two dimensions of workfare on various dimensions of labour market performance. We report the results by means of simulations of the model. To emphasize the wage effect of workfare, we present the results allowing for a decomposition between the total equilibrium effect (termed the full model), and when the wage is kept fixed. This can be interpreted as a decomposition of the total equilibrium effect into a search and a wage component, cf. the effects of workfare outline above.

In the spirit of Fredriksson & Holmlund (2005), we let the instantaneous utility for type $i = e, u, a$ be given by

$$u_i = \ln c_i + \ln f_i$$

where c denotes consumption and f denotes leisure. Specifically, the utility functions for the three types of agents amount to:

$$\begin{aligned} u_e &= \ln dw + \ln(1 - l_e) \\ u_u &= \ln bw + \ln(1 - s_u) \\ u_a &= \ln bw + \ln(1 - s_a - l_a). \end{aligned}$$

where $d > 1$ is a non-monetary return to employment. Unemployment insurance benefits are proportional to the wage and represented by the replacement ratio $b < 1$.

Again, following among others Fredriksson & Holmlund (2001, 2005) the matching function is assumed to be Cobb-Douglas of the form $m = s^\eta v^{1-\eta}$, with $\eta = 0.5$. Also in the tradition of the search literature we impose the Hosios-condition (Hosio, 1990) and set $\beta = \eta = 0.5$.

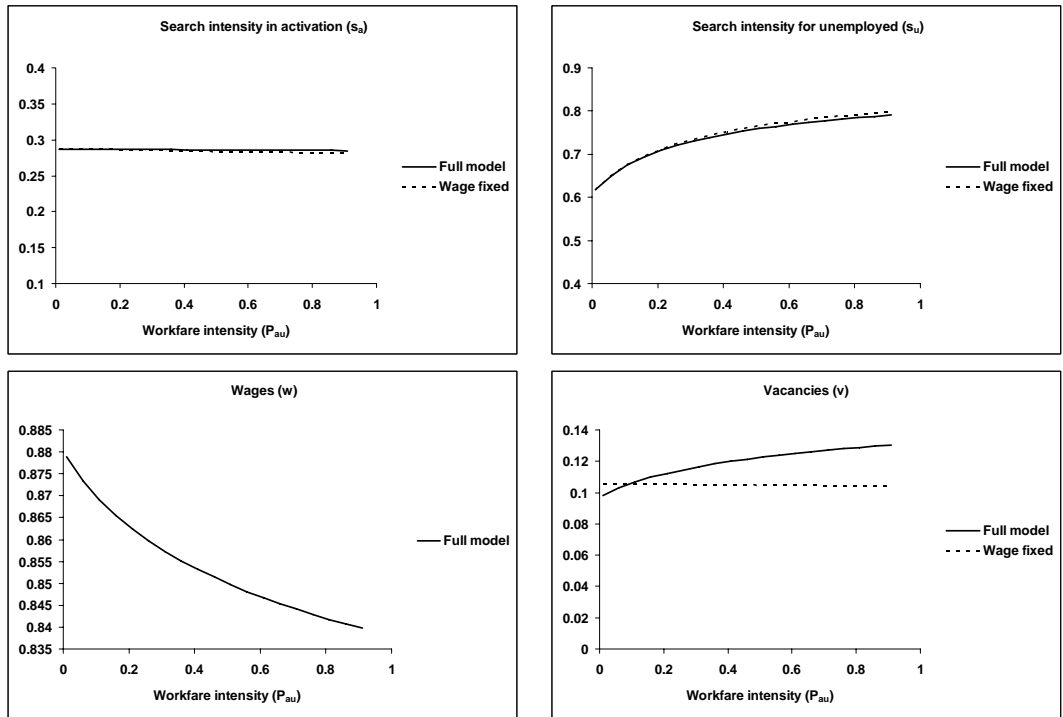
To fit the feature of the Danish labour market we set public consumption, $r = 0.25$, which corresponds to public expenditure of around 30% of GDP and the cost of activating unemployed in workfare programmes, $c = 0.025$, which corresponds to around 3% of GDP. Unemployment insurance in Denmark is relatively generous and to accommodate this the replacement rate is set to $b = 0.6$. We discount utility at $r = 0.01$ and assume that workers spend 60% of their time at work, $l_e = 0.6$. The exogenous exit rate from employment, $p_{ue} = 0.07$, is set to fit the unemployment rate at around 8% in Denmark in the period before increased use of workfare programmes (see e.g. Andersen & Svarer, 2007). Finally, output is set to $y = 1$, vacancy costs are set to $k = 1$ and $d = 4$.

We have conducted simulations for a wide range of parameter values and the qualitative results are in most cases not dependent on the particular parameter choices. In the following it will be pointed out which results are robust to parameters variations, and which are sensitive.

4.1 Intensity of workfare (p_{ua})

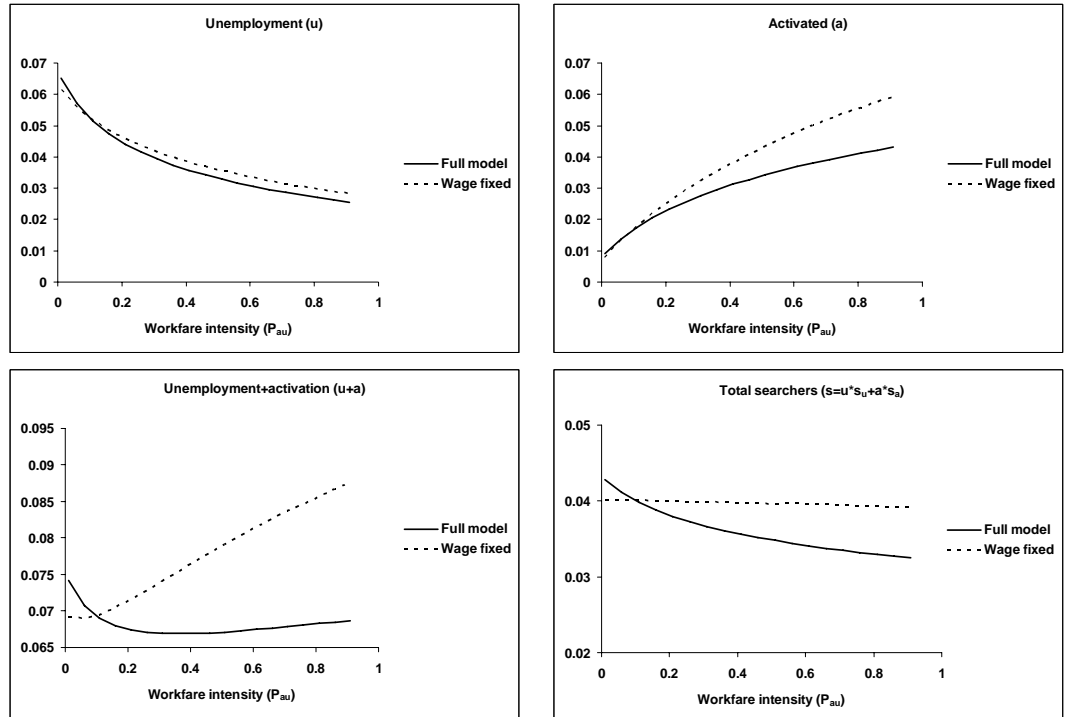
We start by considering the intensity of workfare, ie. the probability by which unemployed are required to participate in some activation measure to remain eligible for unemployment benefits. We assume for the moment that workfare requirement corresponds to full time employment ($l_a = l_e = 0.6$). In Figure 1, we show that there is both a locking-in effect, since activated spend less time searching for jobs than unemployed, and a threat effect, since the search activity for the unemployed increases in the workfare intensity. Notice, that aligned with expression (11) the search intensity for the activated is decreasing in workfare intensity due to the lower expected value of getting a job. Search intensity is basically similar across the simulations with fixed and flexible wages which reflects that the drop in wages is accompanied by a corresponding drop in benefits. Lower wages and a higher search intensity by the unemployed increase the incentive for employers to create jobs and the vacancy rate increases. As the search intensities are basically unaffected by the wage drop, the vacancy increase is accordingly mainly driven by the possibility for employers to earn a higher profit per vacancy since the wage cost is reduced. Also notice, that wages are, due to the market power of firms (search frictions and wage bargaining), lower than marginal productivity.

Figure 1: Effect of workfare intensity on search, wages and vacancies



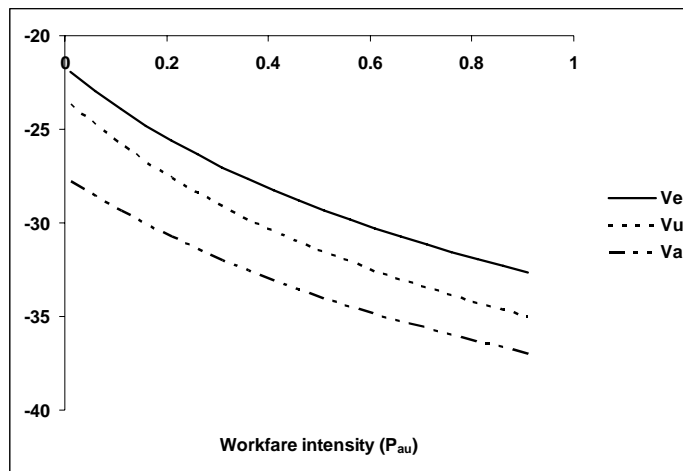
In Figure 2, we see that the increased number of vacancies lower unemployment and that the drop in unemployment is larger when the wage effect is included. Not surprisingly, the number of activated increase and this more so for the model without the wage effect and hence the increase in vacancies that absorb a larger fraction of the potential workfare clients. Considering total unemployment, we find that increasing the intensity of workfare from a low level will lower total unemployment, while if the intensity is high a further increase may increase total unemployment marginally. That is, at a low intensity of workfare the threat and wage effect dominates, while at higher levels the locking-in effect dominates. The number of total searchers is shown to decrease for the particular values of the parameter values. This picture could very well turn around if the workfare requirement was lowered leaving more time to search for the activated.

Figure 2: Effect of workfare intensity on labour market status



Note that since the wage decreases it follows that the economic net-gain from finding a job is reduced, and yet unemployment falls. The reason is that workfare makes claiming of benefits less attractive. Considering welfare we have that the pay-offs in all three labour market states develop similarly, that is, the distributional profile is not much changed, cf. Figure 3.

Figure 3: Pay-offs in labour market states: employment, unemployment and activation

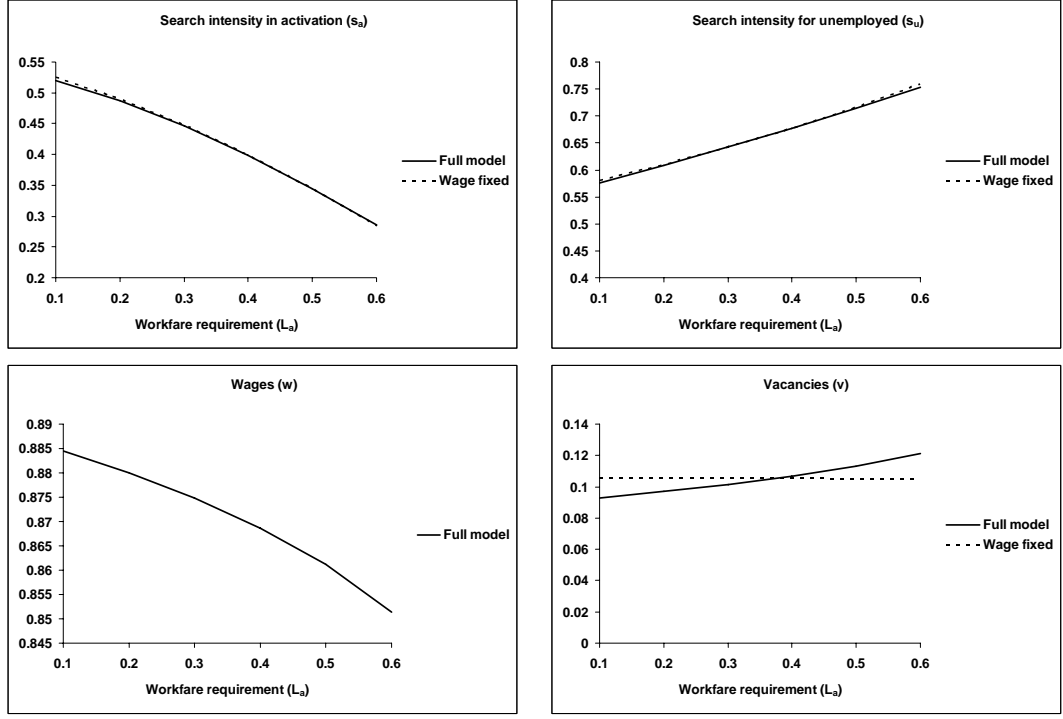


Finally, note that the locking in effect of workfare on the search effort of the activated and the fact that more people are in activation are both poor indicators of the direction in which unemployment (open and total) moves due to workfare policies. This in turn points to the problems in assessing labour market policies from a partial perspective. The same applies when relating total search to unemployment.

4.2 Work requirement in workfare (l_a)

In this section, the workfare intensity is fixed at 0.46 and we show how increasing the work requirement affects various labour market outcomes. In Figure 4 it is shown that increasing the workfare requirement lowers search activity for the activated due to the locking-in effect and increases the search intensity for the unemployed due to the threat effect. Whereas, the latter effect follows unambiguously from the model presented earlier the former could, for other parameter values, have shown a picture where the threat effect present (also) for the activated could have dominated the locking-in effect. Including the wage effect, we find that the deterioration of the outside option of workers cause a reduction in the wage which induce more vacancies.

Figure 4: Effects of workfare requirement on search intensity, wages and vacancies



As shown in Figure 5, the increase in vacancies alongside the more intensive search effort by the unemployed lowers open unemployment. The number of activated increases do to the locking-in effect. On the other hand, there is also a threat effect for those in activation due to the reduced return to unemployment from the increased workfare requirement. As firms post more vacancies in the model with flexible wages this implies that the number of activated initially decreases. Eventually as the workfare requirement increases the locking-in effect dominates the threat effect and activation increase in magnitude.

In Figure 5 total unemployment decreases monotonically as a function of workfare requirement in the model with flexible wages. To take a closer look at this effect the following expression shows the derivative of total unemployment wrt. to the workfare policy variables:

$$\frac{\partial(u+a)}{\partial z} = \frac{\left[\frac{\alpha(s_u - s_a)}{\alpha s_a} \right] u \frac{\partial p_{au}}{\partial z} - \left[1 + \frac{p_{au}}{\alpha s_a} \right] u \frac{\partial \alpha s_u}{\partial z} - \left[\frac{\alpha s_u + p_{au}}{\alpha s_a} \right] a \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}}, \quad z = l_a, p_{au}.$$

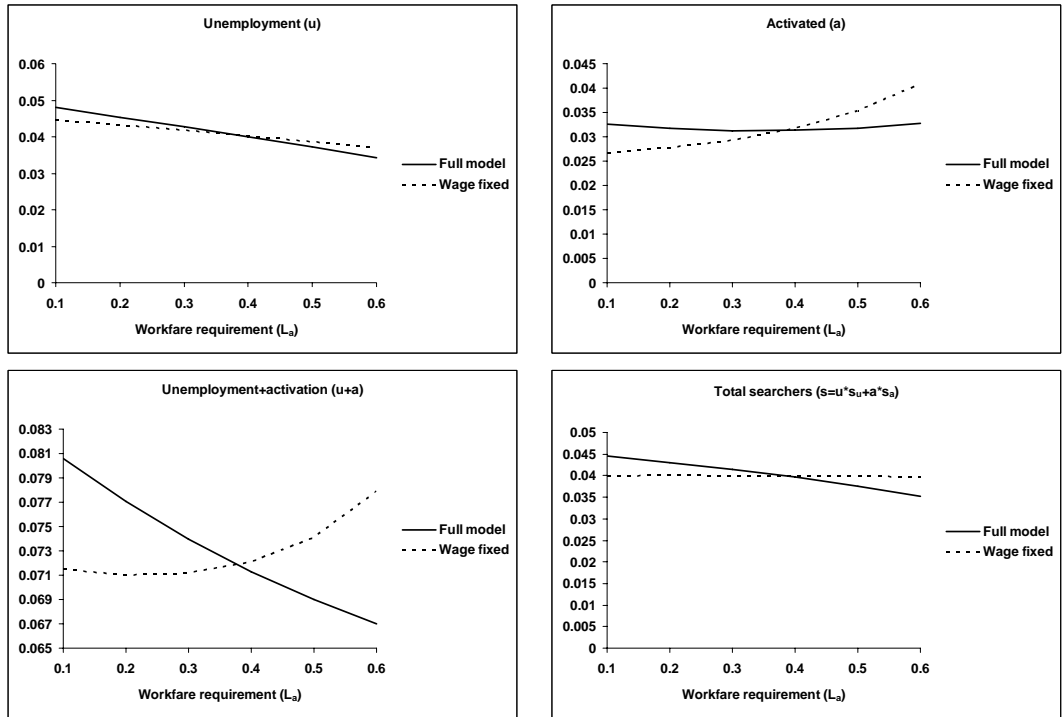
We have already discussed that is not possible to sign this expression. It is however fruitful to decompose the different components and determine their impact on total unemployment. The first term in the numerator is only released by a change in the intensity and goes in the direction of increasing total unemployment if search activity is larger for unemployed than activated. This is

the direct effect of shifting between two groups with different search intensities. More interesting are the next two terms involving the effective job-finding rate (αs) for unemployed and activated respectively. The effective job-finding rate is thus affected by both the wage effect (via α) and the search effect (s), cf (17). If effective job finding rates change in the same direction for the two groups, these two components work in the direction of reducing total unemployment. The above suggest that total unemployment may be non-monotonously related to search intensity as also found in the simulations.

In Figure 2 we found an u-shaped total unemployment rate as a function of workfare intensity. varying the intensity of workfare the search effort for unemployed increases (threat effect) while it is almost constant for the activated. The wage is decreasing and therefore the effective job finding rate is increasing in the intensity of workfare for both groups and this goes in the direction of decreasing total unemployment. As the intensity of workfare increases the difference in search activity for the two groups widens, and this goes in the direction of increasing total unemployment. This accounts for the U-shaped relationship, at low levels of the intensity the effect on the effective job-finding rates dominates, while at higher levels of intensity the difference in search activities dominates.

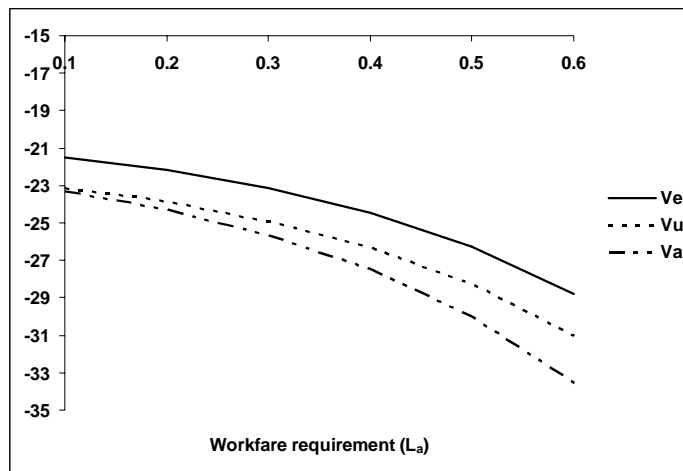
In Figure 5 where p_{au} is fixed and therefore the first term in the expression above is neutralized. Here search activity for unemployed increases (threat effect) while it decreases for the activated (locking-in effect). The wage effect works to increase job finding rates. Hence, effective job finding rates increase for unemployed but it is ambiguous whether it is increasing or decreasing for the activated. In the simulation reported the former dominates and total unemployment is decreasing in the workfare requirement. Clearly this finding is not general, since we have two counteracting effects, and hence a non-monotone relationship may arise.

Figure 5: Effects of workfare requirement on labour market status



If the simulations presented here were taken at face value the recommendation to policy makers that wish to maximize GDP and hence minimize total unemployment is to have intermediate levels of workfare intensity, but strict workfare requirements. Ignoring the wage effect the suggestion would be to have relatively low intensity and low requirement. It is not surprising that including the wage effects make workfare more attractive. Whether policies should have more strict in terms of workfare requirement than workfare intensity is hard to generalize. It appears well established empirically that workfare has a strong locking-in effect $\frac{\partial s_a}{\partial l_a} < 0$ (see e.g. Heckman et al. (1999) and Kluge (2006)) and this suggest that caution should be taken in terms of having to high workfare intensity. Increasing workfare requirements can thus be sufficient to generate wage effects and hence to increase job creation and potentially to increase the effective job finding rate leaving room for medium levels of workfare intensity. There is, however, a relevant counter argument working in the direction of not having to tough workfare requirements. This is given in Figure 6. Here we see that changes in the workfare requirement have the same qualitative effects on the distribution profile for income as a change in the intensity of workfare. However, for the total utility stream or pay-off those on workfare are more negatively affected than the employed and the unemployed.

Figure 6: Pay-offs in labour market states: employment, unemployment and activation



5 Concluding remarks

The present equilibrium search model has shown that workfare releases a locking-in, threat and a wage effect, i.e. it affects the position of all three groups in the labour market (the activated, the unemployed and the employed). Empirical assessments of workfare policies tend to focus on the search effects, but the present analysis shows that the wage effect is crucial for the effects.

It was found that a change in workfare - both the intensity and the work requirement - may shift the trade-off between insurance and incentives in the labour market. In the analysis benefits were kept constant, and it was shown that workfare could be used to improve the incentive structure creating more jobs and lower (open and total) unemployment. It is also an implication of the analysis that partial results - theoretical and empirical - of the effects of workfare policies may be a poor metric for the overall effects due to the interplay between the three effects of workfare policies.

An important topic for future research is to analyse the optimal design of workfare policies. A question which is complicated by the fact that it only gets interesting if taking distributional concerns seriously. In this context it would be interesting to include different types of workers with different types of unemployment risks, since this is an important aspect for policy design.

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Appendix

A: Utility gains

We have from (1), (2) and (3)

$$\begin{aligned}(r + p_{ue} + \alpha s_u)(V^E - V^U) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + p_{au}(V^U - V^A) \\ (r + \alpha s_a)(V^E - V^A) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a) + p_{ue}(V^U - V^E)\end{aligned}$$

which can be written

$$\begin{aligned}(r + p_{ue} + \alpha s_u)(V^E - V^U) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + p_{au}(V^U - V^E + V^E - V^A) \\ (r + \alpha s_a)(V^E - V^A) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a) + p_{ue}(V^U - V^E)\end{aligned}$$

yielding

$$\begin{aligned}(r + p_{ue} + p_{au} + \alpha s_u)(V^E - V^U) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + p_{au}(V^E - V^A) \\ (r + \alpha s_a)(V^E - V^A) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a) + p_{ue}(V^U - V^E)\end{aligned}$$

It follows that

$$(V^E - V^U) = \frac{h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + \frac{p_{au}[u(\tilde{w}, 1 - l_e) - v(b, 1 - s_a - l_a)]}{r + \alpha s_a}}{r + p_{ue} + p_{au} + \alpha s_u + \frac{p_{au}p_{ue}}{r + \alpha s_a}} \quad (18)$$

$$(V^E - V^A) = \frac{h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a) - \frac{p_{ue}[u(\tilde{w}, 1 - l_e) - v(b, 1 - s_u)]}{r + p_{ue} + p_{au} + \alpha s_u}}{r + \alpha s_a + \frac{p_{ue}p_{au}}{r + p_{ue} + p_{au} + \alpha s_u}} \quad (19)$$

B: Marginal rates of substitution

Consider combinations of the transition probability (p_{au}) and work requirement (l_a) leaving the utility gain of employed relative to unemployed unchanged and the utility gain of those on unemployment benefits relative to those on workfare unchanged, i.e.

$$\begin{aligned}V^E - V^U &= \text{constant} \\ V^E - V^A &= \text{constant}\end{aligned}$$

or

$$d(V^E - V^U) = 0 = d(V^E - V^A)$$

we have from (18)

$$\begin{aligned}&\left(1 + \frac{p_{ue}}{r + \alpha s_a}\right)(V^E - V^U) dp_{au} \\ &= \frac{dp_{au}}{r + \alpha s_a} [h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a)] + \frac{p_{au}}{r + \alpha s_a} v'_F(b, 1 - s_a - l_a) dl_a\end{aligned}$$

$$\begin{aligned}
& \frac{p_{au}}{r + \alpha s_a} g'_F(b, 1 - s_a - l_a) dl_a \\
= & \left[\left(1 + \frac{p_{ue}}{r + \alpha s_a} \right) (V^E - V^U) - \frac{1}{r + \alpha s_a} [h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a)] \right] dp_{au}
\end{aligned}$$

Using that

$$(r + \alpha s_a)(V^E - V^A) - p_{ue}(V^U - V^E) = h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a)$$

we get

$$\begin{aligned}
& \frac{p_{au}}{r + \alpha s_a} g'_F(b, 1 - s_a - l_a) dl_a \\
= & \left[\left(1 + \frac{p_{ue}}{r + \alpha s_a} \right) (V^E - V^U) - \frac{1}{r + \alpha s_a} [(r + \alpha s_a)(V^E - V^A) - p_{ue}(V^U - V^E)] \right] dp_{au} \\
= & [(V^A - V^U)] dp_{au}
\end{aligned}$$

Hence

$$\frac{dp_{au}}{dl_a} \Big|_{U=} = -\frac{p_{au}}{r + \alpha s_a} \frac{g'_F(b, 1 - s_a - l_a)}{V^U - V^A} < 0$$

Similarly we have from (19)

$$\begin{aligned}
& \left[\frac{p_{ue}(r + p_{ue} + p_{au} + \alpha s_u) - p_{ue}p_{au}}{(r + p_{ue} + p_{au} + \alpha s_u)^2} \right] (V^E - V^A) dp_{au} \\
= & g'_F(b, 1 - s_a - l_a) dl_a + \frac{p_{ue}}{(r + p_{ue} + p_{au} + \alpha s_u)^2} [h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u)] dp_{au} \\
& [(r + p_{ue} + \alpha s_u)(V^E - V^A) - [h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u)]] \frac{p_{ue}}{(r + p_{ue} + p_{au} + \alpha s_u)^2} dp_{au} \\
= & g'_F(b, 1 - s_a - l_a) dl_a
\end{aligned}$$

using that

$$(r + p_{ue} + p_{au} + \alpha s_u)(V^E - V^U) = h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + p_{au}(V^E - V^A)$$

we get

$$\begin{aligned}
& [(r + p_{ue} + \alpha s_u + p_{au})(V^E - V^A) - (r + p_{ue} + p_{au} + \alpha s_u)(V^E - V^U)] \frac{p_{ue}}{(r + p_{ue} + p_{au} + \alpha s_u)^2} dp_{au} \\
= & g'_F(b, 1 - s_a - l_a) dl_a \\
& - (V^A - V^U) \frac{p_{ue}}{r + p_{ue} + p_{au} + \alpha s_u} dp_{au} = g'_F(b, 1 - s_a - l_a) dl_a
\end{aligned}$$

Hence

$$\frac{dp_{au}}{dl_a} \Big|_A = \frac{r + p_{ue} + p_{au} + \alpha s_u g'_F(b, 1 - s_a - l_a)}{p_{ue}} \frac{g'_F(b, 1 - s_a - l_a)}{V^U - V^A} > 0$$

C: Impact effects of changes in workfare policies

To see the effects of workfare policies it is useful to consider the impact effects of changes in the two elements of workfare, namely the transition probability (p_{au}) and work requirement (l_a) on the utility gains for given macrovariables (w, α, τ). We have

$$\begin{aligned} (r + p_{ue} + p_{au} + \alpha s_u) (V^E - V^U) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + p_{au} (V^E - V^A) \\ (r + \alpha s_a) (V^E - V^A) &= h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a) + p_{ue} (V^U - V^E) \end{aligned}$$

Hence

$$\begin{aligned} \frac{\partial (V^E - V^U)}{\partial l_a} &= \frac{p_{au}}{r + p_{ue} + p_{au} + \alpha s_u} \frac{\partial (V^E - V^A)}{\partial l_a} \\ \frac{\partial (V^E - V^A)}{\partial l_a} &= \frac{1}{r + \alpha s_a} g'_F(b, 1 - s_a - l_a) - \frac{p_{ue}}{r + \alpha s_a} \frac{\partial (V^E - V^U)}{\partial l_a} \\ \frac{\partial (V^E - V^U)}{\partial p_{au}} &= \frac{1}{r + p_{ue} + p_{au} + \alpha s_u} \left[(V^U - V^A) + p_{au} \frac{\partial (V^E - V^A)}{\partial p_{au}} \right] \\ \frac{\partial (V^E - V^A)}{\partial p_{au}} &= - \frac{p_{ue}}{r + \alpha s_a} \frac{\partial (V^E - V^U)}{\partial p_{au}} \end{aligned}$$

Hence, we have

$$\begin{aligned} \frac{\partial (V^E - V^U)}{\partial l_a} > 0 & \quad \frac{\partial (V^E - V^U)}{\partial p_{au}} > 0 \\ \frac{\partial (V^E - V^A)}{\partial l_a} > 0 & \quad \frac{\partial (V^E - V^A)}{\partial p_{au}} < 0 \end{aligned}$$

D: Equilibrium

The model can be summarized by the following 10 equations in the following endogenous variables: $V^E - V^U, V^E - V^A, s_a, s_u, a, u, w, q, \alpha, \theta$.

Pay-off gains:

$$V^E - V^U = \frac{h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_u) + p_{au} (V^U - V^A)}{r + p_{ue} + \alpha s_u} \quad (20)$$

$$V^E - V^A = \frac{h(w[1 - \tau], 1 - l_e) - g(b, 1 - s_a - l_a) + p_{ue} (V^U - V^E)}{r + \alpha s_a} \quad (21)$$

Search effort:

$$g'_F(b, 1 - s_u) = \alpha [V^E - V^U] \quad (22)$$

$$g'_F(b, 1 - s_a - l_a) = \alpha [V^E - V^A] \quad (23)$$

Inflow and outflows:

$$[1 - u - a] p_{ue} = \alpha(\theta) [s_u u + s_a a] \quad (24)$$

$$\alpha(\theta) s_a a = p_{au} u \quad (25)$$

Job creation and wage setting:

$$w = \left[1 - (r + p_{ue}) \frac{k}{q} \right] y \quad (26)$$

$$0 = \beta \frac{u_w(w(1-\tau), 1-l_e)}{V^E - V^U} - (1-\beta) \frac{q}{ky} \quad (27)$$

Job-finding and Job-filling rates:

$$\alpha = \frac{m(s, v)}{s} = m(1, \theta), m_\theta > 0 \quad (28)$$

$$q = \frac{m(s, v)}{v} = m(\theta^{-1}, 1), q'(\theta) < 0; \theta = \frac{v}{s} \quad (29)$$

Using (26), (28) and (29) we have

$$q = \tilde{q}(w) \quad \frac{\partial \tilde{q}(w)}{\partial w} > 0$$

$$\alpha = \tilde{\alpha}(w) \quad \frac{\partial \tilde{\alpha}(w)}{\partial w} < 0$$

$$\theta = \tilde{\alpha}(w) \quad \frac{\partial \tilde{\theta}(w)}{\partial w} < 0$$

Search activities can from (22) and (22) be written

$$\begin{aligned} s_u &= \Phi(\tilde{\alpha}(w) [V^E - V^U]) \\ s_a &= \Phi(\tilde{\alpha}(w) [V^E - V^A]) - l_a \end{aligned}$$

and using this in (20) and (21) it follows that utility gains can be written

$$\begin{aligned} V^E - V^U &= F(w, \tau, \tilde{\alpha}(w), p_{au}, l_a) \\ V^E - V^A &= G(w, \tau, \tilde{\alpha}(w), p_{au}, l_a) \end{aligned}$$

Finally, the wage equation (26) can now be written

$$\beta \frac{u_w(w(1-\tau), 1-l_e)}{V^E - V^U} - (1-\beta) \frac{q(\theta)}{ky} = 0$$

or¹²

$$\begin{aligned} 0 &= \Psi(w, \tau, \tilde{q}(w), V^E - V^U) \quad \Psi_w < 0, \Psi_{V^E - V^U} < 0 \\ &= \Psi(w, \tau, \tilde{q}(w), F(w, \tau, \tilde{\alpha}(w), p_{au}, l_a)) \end{aligned} \quad (30)$$

For a given tax rate τ the equilibrium wage is found as the solution to (30). If the function Ψ is monotonously decreasing in the wage rate, it follows that

¹²Where Ψ_w follows from the second order condition.

the equilibrium is unique. This implies that $\frac{\partial w}{\partial p_{au}} < 0$, $\frac{\partial w}{\partial l_a} < 0$. Note that an equilibrium where $0 < u + a < 1$ is ensured since if $u + a = 0$ we $u = a = 0$ implying that $\tilde{\alpha}(w) = p_{ue}$ which is inconsistent with (29), and for $u + a = 1$ we have $\tilde{\alpha}(w) = 0$ which is also inconsistent with (29)

Note that endogenizing the tax rate would introduce a non-linearity in the model which potentially could imply multiple equilibria since

$$\tau = \frac{bu + (b + c)a + r}{w(1 - u - a)}$$

For this reason the tax rate is assumed constant in the theoretical analysis, but the tax rate is endogenized in the numerical examples.

E: Effects of changes in workfare policies

In this appendix we consider how changes in workfare policies (p_{au}, l_a) affect unemployment (u) , activation (a) and total unemployment $(u + a)$.

We have from (12) and (13) that

$$\begin{aligned} [1 - u - a] p_{ue} &= \alpha [s_u u + s_a a] \\ \alpha s_a a &= p_{au} u \end{aligned}$$

Hence,

$$\begin{aligned} 0 &= (p_{ue} + \alpha s_u) \frac{\partial u}{\partial z} + (p_{ue} + \alpha s_a) \frac{\partial a}{\partial z} + u \frac{\partial \alpha s_u}{\partial z} + a \frac{\partial \alpha s_a}{\partial z} \\ \alpha s_a \frac{\partial a}{\partial z} + a \frac{\partial \alpha s_a}{\partial z} &= p_{au} \frac{\partial u}{\partial z} + u \frac{\partial p_{au}}{\partial z} \end{aligned}$$

and it follows that

$$\begin{aligned} 0 &= (p_{ue} + \alpha s_u) \frac{\partial u}{\partial z} + \frac{p_{ue} + \alpha s_a}{\alpha s_a} \left[p_{au} \frac{\partial u}{\partial z} + u \frac{\partial p_{au}}{\partial z} - a \frac{\partial \alpha s_a}{\partial z} \right] + u \frac{\partial \alpha s_u}{\partial z} + a \frac{\partial \alpha s_a}{\partial z} \\ \frac{\partial u}{\partial z} &= \frac{-\frac{p_{ue} + \alpha s_a}{\alpha s_a} u \frac{\partial p_{au}}{\partial z} - u \frac{\partial \alpha s_u}{\partial z} + a \frac{p_{ue}}{\alpha s_a} \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}} \end{aligned}$$

Considering the effect on activation we have

$$\begin{aligned} \frac{\partial a}{\partial z} &= \frac{1}{\alpha s_a} \left[p_{au} \frac{\partial u}{\partial z} + u \frac{\partial p_{au}}{\partial z} - a \frac{\partial \alpha s_a}{\partial z} \right] \\ &= \frac{1}{\alpha s_a} \left[\frac{p_{au} \left[-\frac{p_{ue} + \alpha s_a}{\alpha s_a} u \frac{\partial p_{au}}{\partial z} - u \frac{\partial \alpha s_u}{\partial z} + a \frac{p_{ue}}{\alpha s_a} \frac{\partial \alpha s_a}{\partial z} \right] + \left[(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au} \right] \left[u \frac{\partial p_{au}}{\partial z} - a \frac{\partial \alpha s_a}{\partial z} \right]}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}} \right] \\ &= \frac{1}{\alpha s_a} \left[\frac{(p_{ue} + \alpha s_u) u \frac{\partial p_{au}}{\partial z} - u p_{au} \frac{\partial \alpha s_u}{\partial z} - [p_{ue} + \alpha s_u + p_{au}] a \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}} \right] \end{aligned}$$

and combining the two the effect on total unemployment is found to be

$$\frac{\partial u}{\partial z} + \frac{\partial a}{\partial z} = \frac{\left[\frac{\alpha(s_u - s_a)}{\alpha s_a} \right] u \frac{\partial p_{au}}{\partial z} - \left[1 + \frac{p_{au}}{\alpha s_a} \right] u \frac{\partial \alpha s_u}{\partial z} - \left[\frac{\alpha s_u + p_{au}}{\alpha s_a} \right] a \frac{\partial \alpha s_a}{\partial z}}{(p_{ue} + \alpha s_u) + \frac{p_{ue} + \alpha s_a}{\alpha s_a} p_{au}}$$

In the following we consider the incremental introduction of workfare policies into a benefit scheme without workfare. We neutralize the unambiguous wage effect, and focus on the search effects which in general are ambiguous, to show that they are unambiguous for an incremental introduction of workfare. The question addressed is thus whether starting with a benefit scheme without workfare elements a marginal introduction of workfare will lower unemployment (both open and total, i.e. u and $u + a$). This can happen in one of two ways, either having an initial situation where $(p_{au}, l_a) = (0, l_a)$ and then rising p_{au} marginally, or having an initiation situation $(p_{au}, l_a) = (p_{au}, 0)$ and then rising l_a marginally. The wage rate w and thus job finding rate α are constant to focus on the search effect.

(I) Incidence of workfare

Note that for $p_{au} = 0$ we have $a = 0$, and hence

$$\begin{aligned} \alpha s_a \frac{\partial a}{\partial p_{au}} \Big|_{p_{au}=0} &= u \\ (p_{ue} + \alpha s_u) \frac{\partial u}{\partial p_{au}} \Big|_{p_{au}=0} &= -(p_{ue} + \alpha s_a) \frac{\partial a}{\partial p_{au}} \Big|_{p_{au}=0} + u \alpha \frac{\partial s_u}{\partial p_{au}} \Big|_{p_{au}=0} \end{aligned}$$

Implying

$$\frac{\partial u}{\partial p_{au}} \Big|_{p_{au}=0} = - \frac{(p_{ue} + \alpha s_a) \frac{u}{\alpha s_a} + u \alpha \frac{\partial s_u}{\partial p_{au}} \Big|_{p_{au}=0}}{(p_{ue} + \alpha s_u)} < 0$$

where $\frac{\partial s_u}{\partial p_{au}} \Big|_{p_{au}=0} > 0$ follows from (10).

$$\frac{\partial(u+a)}{\partial p_{au}} \Big|_{p_{au}=0} = - \frac{u \alpha \frac{\partial s_u}{\partial p_{au}} \Big|_{p_{au}=0}}{(p_{ue} + \alpha s_u)} + \left[1 - \frac{(p_{ue} + \alpha s_a)}{(p_{ue} + \alpha s_u)} \right] \frac{u}{\alpha s_a}$$

Note that this is negative for $l_a = 0$, hence, there exists a \bar{l}_a such that $\frac{\partial(u+a)}{\partial p_{au}} \Big|_{p_{au}=0} < 0$.

(II) work requirement

Note that if $l_a = 0$ agents are similarly situated as unemployed and in activation ($s_a = s_u$) and we have

$$\begin{aligned} 0 &= (p_{ue} + \alpha s_u) \left[\frac{\partial u}{\partial l_a} + \frac{\partial a}{\partial l_a} \right] + u \alpha \frac{\partial s_u}{\partial l_a} + a \alpha \frac{\partial s_a}{\partial l_a} \\ \alpha s_a \frac{\partial a}{\partial l_a} + a \alpha \frac{\partial s_a}{\partial l_a} &= p_{au} \frac{\partial u}{\partial l_a} \end{aligned}$$

hence

$$0 = (p_{ue} + \alpha s_u) \frac{\partial u}{\partial l_a} + \frac{(p_{ue} + \alpha s_u)}{\alpha s_a} \left[p_{au} \frac{\partial u}{\partial l_a} - a \alpha \frac{\partial s_a}{\partial l_a} \right] + u \alpha \frac{\partial s_u}{\partial l_a} + a \alpha \frac{\partial s_a}{\partial l_a}$$

or

$$\left[(p_{ue} + \alpha s_u) + \frac{(p_{ue} + \alpha s_u)}{\alpha s_a} p_{au} \right] \frac{\partial u}{\partial l_a} = \left[\frac{(p_{ue} + \alpha s_u)}{\alpha s_a} - 1 \right] a \alpha \frac{\partial s_a}{\partial l_a} - u \alpha \frac{\partial s_u}{\partial l_a}$$

We have $\frac{(p_{ue} + \alpha s_u)}{\alpha s_a} - 1 > 0$ and hence $\frac{\partial u}{\partial l_a} < 0$ has as a sufficient condition $\alpha \frac{\partial s_a}{\partial l_a} < 0$. Note that (8) implies that $\alpha \frac{\partial s_a}{\partial l_a} < 0$. For $p_{au} = 0$ we have $a = 0$, and $\frac{\partial u}{\partial l_a} < 0$ and $\frac{\partial u}{\partial l_a} + \frac{\partial a}{\partial l_a} < 0$. Moreover there is a \bar{p}_{au} such that for $p_{au} < \bar{p}_{au}$ we have $\frac{\partial u}{\partial l_a} < 0$ and $\frac{\partial u}{\partial l_a} + \frac{\partial a}{\partial l_a} < 0$.