Trend Inflation, Taylor Principle and Indeterminacy

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Abstract

In this paper, we show that low trend inflation strongly affects the dynamics of a standard Neo-keynesian model where monetary policy is described by a standard Taylor rule. In particular, we show that trend inflation: (i) enlarges the indeterminacy region in the parameter space, substantially altering the so-called Taylor principle; (ii) changes the dynamic responses of the economy. Furthermore, we generalize the basic analysis to different types of Taylor rules, inertial policy rules and indexation schemes. The key point is that, whatever the set up, the literature on Taylor rules cannot disregard average inflation in both theoretical and empirical analysis.

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1 Introduction

Average inflation in the post-war period in developed countries was moderately different from zero and varied across countries.[1] Nonetheless, most of the vast literature on monetary policy rules worked with models log-linearized around a zero inflation steady state (see e.g., Clarida et al., 1999, Galí, 2003, Woodford, 2003, or the book edited by Taylor, 1999). This paper aims to accomodate this manifest inconsistency.

We generalize a standard Neo-Keynesian model with Calvo staggered price by taking a log-linear approximation around a general level of steady state inflation.[2] Then we use a Taylor rule to close the model and address the question of how the properties of our economy change as the trend inflation level varies.[3]

Our key finding is that trend inflation greatly affects the existing results in the literature. In particular moderate levels of trend inflation: (i) modify the determinacy region in the parameters space; (ii) alter the impulse response function of the model economy after a cost-push shock. As a consequence, trend inflation largely changes also the (unconditional) variances of key variables, such as inflation and output.

With respect to (i), we show that trend inflation substantially changes the well-known determinacy condition that the literature labelled the *Taylor principle.* This result is due to the distortions trend inflation causes in the long-run properties of the model and, particularly, in the steady state relationship between inflation and output, a surprisingly neglected issue in the literature. The long-run Phillips curve is highly non-linear in the Neo-Keynesian model: it is positively sloped when steady state inflation is zero, but then turns quite rapidly negative for extremely low value of trend inflation, because of the strong price-dispersion effect.[4] We will show that this has significant implications on the celebrated Taylor principle. The results in most of the literature

[1] For example, Schmitt-Grohe and Uribe (2004b) calibrate trend inflation for the U.S. to 4.2%, based on data from 1960-1998. In the same period Germany, Italy, Spain, and UK exhibit an average inflation equals to respectively 3.22%, 8.12%, 7.1% and 9% (source: OECD).

[2] In this paper, we abstract for other possible form of frictions, since we want to investigate the relationship between Taylor rules and trend inflation.

[3] In the following analysis we shall use indifferently trend inflation or long-run inflation to denote the level of inflation rate in the deterministic steady state.

[4] See Ascari (1998) and Ascari (2004).

are therefore based on a case (i.e., zero steady state inflation) that is both empirical unrealistic and theoretically very special.

Our key result is then generalized and proved to be robust to: (i) different kinds of Taylor type rules proposed in the literature (contemporaneous, backward-looking, forward-looking and hybrid, see e.g., Clarida et al., 2000, Bullard and Mitra, 2002); (ii) inertial Taylor rules for all the cases in (i); (iii) indexation schemes used in the literature (see, e.g., Yun, 1996 and Christiano et al., 2005); (iv) different parameter values.

In sum, this paper shows that the literature on monetary policy rules cannot neglect trend inflation both in the empirical and theoretical analysis, because the specification of the theoretical model and then all the results are very sensitive to low and moderate trend inflation levels, as empirically observed in western countries.

Just to give an example, the seminal analysis in Clarida et al. (2000) can be misleading. Indeed, Clarida et al. (2000) data set features an average inflation for the US economy quite different from zero inflation, while their analysis is based on a theoretical model that assumes zero trend inflation. On the one hand, positive trend inflation changes the determinacy region, and then the well-known Taylor principle, such that in order to label the equilibrium determinate one needs to take trend inflation into account. On the other hand, once an equilibrium is identified to pass from determinate to indeterminate or vice versa, it is still to be investigated what is the relative contribution of a change in the monetary policy regime (i.e, a change in the Taylor rule parameters) rather than a change in the trend inflation level.

Another contribution of the paper is to offer a detailed presentation of the standard log-linear Neo-Keynesian model approximated around a general trend inflation level with and without indexation schemes. As such the paper generalizes the model in Ascari and Ropele (2004) allowing for indexation schemes, and complements a series of recent papers. Indeed, only very few papers in the literature allow for positive trend inflation level. Ascari (2004) illustrates a standard Neo-keynesian model log-linearised around a general trend inflation level. Ascari and Ropele (2004) analyzes how optimal short-run monetary policy changes with trend inflation. Khan et al. (2003) instead solves the optimal monetary policy problem and then investigate the dynamics of the economy around the given optimal steady state inflation level. Schmitt-Grohe and

Uribe (2004a,b) simulates the model under different Taylor type rules calibrating average inflation on US data, but it does not study the effects of changing trend inflation levels. Moreover, Schmitt-Grohe and Uribe (2004a,b) allows for the indexation scheme proposed by Christiano, Eichenbaum, and Evans (2005), but simulates the model up to second-order, so that the model is not log-linearised.

Finally, Kiley (2004) is a very related paper to ours. Kiley (2004) investigates the effect of trend inflation in a model where prices are staggered a là Taylor (1979) and monetary policy is described by Taylor rules.[5] Our paper also complements this very recent paper by assuming the more popular Calvo (1983) staggered pricing framework, and by generalizing the results to different Taylor type rules and indexation schemes.

The paper proceeds as follows. Section 2 presents the model and Section 3 displays the log-linearised version of it. Section 4 then presents the main results of the paper, by looking at the behavior of the model when monetary policy is described by a contemporaneous Taylor rule. Section 5 tests the robustness of our key findings to many alternative assumptions, as illustrated above. Section 6 concludes.

2 The Model

In this section we describe a simple New Keynesian stochastic general equilibrium model, similar to Clarida, Galí, and Gertler (1999), Galí (2003) and Woodford (2003), generalized to allow for positive trend inflation (as in Ascari, 2004) and indexation.

2.1 Households

The economy is populated by infinitively lived households whose instantaneous utility function is increasing in the consumption of the final good (C_t) and real money balances (M_t/P_t) and decreasing in labor (N_t) according to

$$U\left(C, \frac{M}{P}, N\right) = \frac{C_t^{1-\sigma_c} - 1}{1-\sigma_c} + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \qquad (1)$$

[5] We became aware of Kiley (2004) when we already embarked working on this paper. Kiley (2004) shows that also in Taylor (1979) type of framework, trend inflation influences the determinacy region and the unconditional variance of inflation. Kiley (2004) model is however quite stylized (two-period staggering) and the analysis "kept as simple as possible" (p. 26). We therefore complements and generalizes its results.

where the positive parameters σ_c, σ_m and σ_n represents the inverse of the intertemporal elasticity of substitution in consumption, real money balances and labor supply, respectively, while χ_m and χ_n are positive constants.

At a given period t, the representative household faces the following nominal flow budget constraint

$$P_t C_t + M_t + B_t \leq W_t N_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + F_t + TR_t \tag{2}$$

where P_t is the price of the final good, M_t represents holding of nominal money, B_t represents holding of bonds offering a one-period nominal return i_t, W_t is the nominal wage and F_t are firms profits rebated to the households. In addition, each period the government makes lump-sum nominal transfers to households equal to TR_t. The household's problem is to maximize the lifetime expected utility subject to budget constraints (2), that is

$$\max_{\left\{C_t, \frac{M_t}{P_t}, N_t, B_t\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma_c} - 1}{1-\sigma_c} + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right) \tag{3}$$

$$s.t. \qquad C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{F_t}{P_t} + \frac{TR_t}{P_t}$$

where $\beta \in (0, 1)$ is the subjective rate of time preference and E_0 denotes the expectation operator conditional on the time $t = 0$ information set. The resulting first order conditions yield:

$$\textit{labor supply} \quad : \qquad \chi_n \frac{N_t^{\sigma_n}}{C_t^{-\sigma_c}} = \frac{W_t}{P_t} \tag{4}$$

$$\textit{money demand} \quad : \qquad \chi_m \frac{(M_t/P_t)^{-\sigma_m}}{C_t^{-\sigma_c}} = \frac{i_t}{1+i_t} \tag{5}$$

$$\textit{consumption Euler eq.} \quad : \qquad 1 = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma_c}}{C_t^{-\sigma_c}} (1 + i_t) \frac{P_t}{P_{t+1}} \right\}. \tag{6}$$

(4), (5), (6) have the usual straightforward economic interpretation.[6]

[6]Note that the momentary utiltiy function is additively separable in all the three arguments, consumption, real money balances and labor, so that it follows that real money balances will not enter in any of the other structural equations of the model. That is, the money demand equation becomes completely recursive to the rest of the system equations.

2.2 Final Good Producers

In each period t, a final good Y_t is produced by perfectly competitive firms, combining a continuum of intermediate inputs $Y_t(i)$, according to the following standard CES production function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}} \qquad \text{with } \theta > 1. \tag{7}$$

Taking prices as given the final good producer chooses the quantities of intermediate goods $Y_t(i)$ that maximize its profits, i.e., $\max_{Y_t(i)} \left\{ P_t \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i)\, Y_t(i)\, di \right\}$, resulting in the following demand function for each intermediate good i:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t. \tag{8}$$

The zero profit condition in the final good sector brings about the following expression for the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta}\, di\right]^{\frac{1}{1-\theta}}. \tag{9}$$

2.3 Intermediate Good Producers

The intermediate inputs $Y_t(i)$ are produced by a continuum of firms indexed by $i \in [0, 1]$, with the following production technology with constant returns to scale to labor:

$$Y_t(i) = N_t(i). \tag{10}$$

The intermediate goods sector is characterized by the fact that prices are sticky. In particular, intermediate good producers act as monopolistic competitors and set prices according to a standard discrete version of the mechanism put forward by Calvo (1983). In each period, there exists a fixed probability $(1-\alpha)$ according to which a firm can re-optimize its nominal price. On the contrary, with probability α the firm cannot set a new price. In the literature, we can distinguish three different hypothesis about what happens to the price in this latter case: (i) No indexation: the price does not change; (ii) Indexation to trend inflation (e.g., Yun (1996)): the price is automatically fully or partially adjusted according to the level of trend inflation; (iii) Indexation to past inflation (e.g., Christiano, Eichenbaum, and Evans (2005))[7]: the price is automatically fully or partially adjusted according to the past inflation level.

1. No indexation

The problem of a price-resetting firm can be formulated as

$$\max_{p_t^*(i)} \quad E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[\frac{p_t^*(i)}{P_{t+j}} Y_{t+j}(i) - TC_{t+j}^r \left(Y_{t+j}(i) \right) \right]$$

$$s.t. \quad Y_{t+j}(i) = \left[\frac{p_t^*(i)}{P_{t+j}} \right]^{-\theta} Y_{t+j}$$

where $p_t^*(i)$ denotes the new optimal price of producer i , $TC_{t+j}^r (Y_{t+j}(i))$ the real total cost function and $\Delta_{t,t+j}$ is the stochastic discount factor. The solution to this problem yields the familiar formula for the standard optimal resetted price in a Calvo's setup

$$p_t^*(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[P_{t+j}^{\theta} Y_{t+j} MC_{t+j}^r(i) \right]}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[P_{t+j}^{\theta-1} Y_{t+j} \right]} \tag{11}$$

where $MC_t^r(i)$ denotes the real marginal costs function, which, given the production function (10), is simply $MC_t^r \equiv \frac{\partial TC(i)}{\partial Y(i)} = \frac{W_t}{P_t}$, and hence equal across producers i. The real marginal costs thus depends only upon aggregate quantities, namely the real wage.

2. Partial indexation to long-run inflation (LRI)

Under this assumption, a firm that cannot re-optimize its price, update the price according to this simple rule:

$$p_t^*(i) = \bar{\pi}^{\varepsilon} p_{t-1}^*(i) \tag{12}$$

where $\bar{\pi}$ is the steady state inflation level and $\varepsilon \in [0,1]$ is a parameter that measures the degree of indexation. If $\varepsilon = 1$ there is full indexation, if $\varepsilon = 0$ there is no indexation and the problem is the same one as in the previous case. The problem then becomes the following

$$\max_{p_t^*(i)} \quad E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[\frac{p_t^*(i)\, \bar{\pi}^{\varepsilon j}}{P_{t+j}} Y_{t+j}(i) - TC_{t+j}^r \left(Y_{t+j}(i) \right) \right]$$

$$s.t. \quad Y_{t+j}(i) = \left[\frac{p_t^*(i)\, \bar{\pi}^{\varepsilon j}}{P_{t+j}} \right]^{-\theta} Y_{t+j}$$

[7] See also Maury and Sahuc (2004).

and the FOC is

$$p_t^*(i) = \frac{\theta}{\theta-1}\frac{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[P_{t+j}^{\theta}Y_{t+j}MC_{t+j}^r(i)\,\bar{\pi}^{-\theta\varepsilon j}\right]}{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[P_{t+j}^{\theta-1}Y_{t+j}\bar{\pi}^{(1-\theta)\varepsilon j}\right]} \tag{13}$$

3. Partial indexation to past inflation (PI)

Under this assumption, a firm that cannot re-optimize its price, update the price according to this simple rule:

$$p_t^*(i) = \pi_{t-1}^{\varepsilon}p_{t-1}^*(i) \tag{14}$$

where $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$ is the inflation level in the previous period and $\varepsilon \in [0,1]$ is again a parameter that measures the degree of indexation. The problem then becomes the following

$$\max_{p_t^*(i)} \quad E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[\frac{p_t^*(i)\,\Pi_{t,t+j-1}}{P_{t+j}}Y_{t+j}(i) - TC_{t+j}^r\left(Y_{t+j}(i)\right)\right]$$

$$s.t. \quad Y_{t+j}(i) = \left[\frac{p_t^*(i)\,\Pi_{t,t+j-1}}{P_{t+j}}\right]^{-\theta}Y_{t+j}$$

where $\Pi_{t,t+j-1} = \pi_t^{\varepsilon}\pi_{t+1}^{\varepsilon}...\pi_{t+j-1}^{\varepsilon} = \prod_{i=0}^{j-1}\pi_{t+1}^{\varepsilon}$for $j > 0$ and equal zero for $j = 0$. The FOC of this problem is

$$p_t^*(i) = \frac{\theta}{\theta-1}\frac{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[P_{t+j}^{\theta}Y_{t+j}MC_{t+j}^r(i)\,\Pi_{t,t+j-1}^{-\theta}\right]}{E_t\sum_{j=0}^{\infty}\alpha^j\Delta_{t,t+j}\left[P_{t+j}^{\theta-1}Y_{t+j}\Pi_{t,t+j-1}^{1-\theta}\right]} \tag{15}$$

2.4 Government

The government injects money into the economy through nominal transfers, such that:

$$TR_t = M_t^s - M_{t-1}^s \tag{16}$$

where M^s is aggregate nominal money supply. Most importantly, we assume that in steady state money supply evolves according to the following fixed rule

$$M_t^s = \bar{\pi}M_{t-1}^s \tag{17}$$

where $\bar{\pi}$ is the (gross) rate of nominal money supply growth, which in steady state coincides with steady state inflation.

As stated in the Introduction, this paper takes the trend inflation rate, $\bar{\pi}$, as exogenous to the model. In the short run, hence, monetary policy aims at stabilizing inflation and output gap around the long-run targets in response to exogenous shocks buffeting the economy. Finally, we assume that monetary policy is implemented through a Taylor-type rule for the control of the short-term nominal interest rate. Therefore, in the subsequent sections, we will use a Taylor-type rule to close the model and thus equation (5) will become redundant.[8]

2.5 Market clearing conditions

The market clearing conditions in the goods markets, in the money market and in the labour market are simply:

$$\begin{aligned} Y_t &= C_t; \qquad Y_t^s(i) = Y_t^D(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t \qquad \forall i \\ M_t &= M_t^s; \qquad \text{and} \qquad N_t = \int_0^1 N_t(i)di. \end{aligned} \tag{18}$$

3 The log-linearized model

Log-linearizing (4) and (6) we obtain

$$\sigma_n \hat{N}_t + \sigma_c \hat{Y}_t = \hat{W}_t - \hat{P}_t \tag{19}$$

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma_c^{-1} \left[\hat{\imath}_t - E_t \hat{\pi}_{t+1}\right] \tag{20}$$

where hatted variables denote percentage deviations from deterministic steady state and $\hat{\imath}_t \equiv \log\left(\frac{1+i_t}{1+\bar{\imath}}\right)$. Moreover, we used the market clearing condition $\hat{Y}_t = \hat{C}_t$ to obtain the standard forward-looking IS curve (20).

3.1 Generalized New Keynesian Phillips Curves

The log-linearization of the equations for optimal price under trend inflation are definitely more cumbersome than the standard NKPC calculated log-linearising (11) around zero inflation. In the appendix we show the following results.

[8] In the usual sense that we will assume that, given equation (5), the money supply follows the path necessary to implement the short-term nominal interest rate dictated by the Taylor-rule.

1. NKPC with no indexation

In the case of no indexation, as in Ascari and Ropele (2004), the log-linearisation of (11) leads to the following equations

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \lambda(\bar{\pi})\hat{mc}_t + \lambda(\bar{\pi})\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})}\hat{Y}_t + \lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\hat{\psi}_t \qquad (21)$$

and

$$\hat{\psi}_t = \left(1-\alpha\beta\bar{\pi}^{\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{mc}_t\right] + \alpha\beta\bar{\pi}^{\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right] \qquad (22)$$

where $\lambda(\bar{\pi}) = \frac{\left(1-\alpha\bar{\pi}^{\theta-1}\right)\left(1-\alpha\beta\bar{\pi}^{\theta}\right).}{\alpha\bar{\pi}^{\theta}}$.

2. NKPC with partial indexation to long-run inflation

In the case of LRI, the log-linearisation of (13) leads to the following equations

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \lambda_{LR}(\bar{\pi})\hat{mc}_t + \lambda_{LR}(\bar{\pi})\frac{\left(1-\bar{\pi}^{(1-\varepsilon)}\right)(1-\sigma_c)}{\left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)}\hat{Y}_t + \lambda_{LR}(\bar{\pi})\left(\frac{\bar{\pi}^{(1-\varepsilon)}-1}{1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}}\right)\hat{\psi}_t \qquad (23)$$

and

$$\hat{\psi}_t = \left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{mc}_t\right] + \alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right]. \qquad (24)$$

where $\lambda_{LR}(\bar{\pi}) = \frac{\left(1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}\right)\left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right).}{\alpha\bar{\pi}^{(1-\varepsilon)\theta}}$. Note that (23) and (24) are the same as (21) and (22) respectively, where simply $\bar{\pi}$ is replaced by $\bar{\pi}^{(1-\varepsilon)}$. Hence, putting $\varepsilon = 0$, one gets back to the previous case of no indexation.

3. NKPC with partial indexation to past inflation

In the case of PI, the log-linearisation of (15) yields the following equations

$$\begin{aligned}\hat{\pi}_t &= \frac{\varepsilon}{1+\beta\varepsilon}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\varepsilon}E_t\hat{\pi}_{t+1} + \lambda_P(\bar{\pi})\hat{mc}_t + \lambda_P(\bar{\pi})\frac{\left(1-\bar{\pi}^{1-\varepsilon}\right)(1-\sigma_c)}{1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}}\hat{Y}_t + \qquad (25)\\ &\quad +\lambda_P(\bar{\pi})\left(\frac{\bar{\pi}^{1-\varepsilon}-1}{1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}}\right)\hat{\psi}_t\end{aligned}$$

and

$$\hat{\psi}_t = \left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{mc}_t\right] + \alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\left[\theta\hat{\pi}_{t+1} - \theta\varepsilon\hat{\pi}_t + \hat{\psi}_{t+1}\right] \qquad (26)$$

where $\lambda_P(\bar{\pi}) = \frac{\left(1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}\right)\left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)}{(1+\beta\varepsilon)\alpha\bar{\pi}^{(1-\varepsilon)\theta}}$.

Looking carefully at the three NKPCs some comments are in order. First, in all the three examples it appears a new driving variable for the dynamics of inflation: $\hat{\psi}_t$. As shown in the Appendix, $\hat{\psi}_t$ is (the log-deviation of) the numerator in the expression for the optimal resetting price, i.e., (11), (15) and (13) respectively. $\hat{\psi}_t$ is therefore the present discounted value of future marginal costs, where the weights used in the discounting depend on future expected output and inflation levels. These weights can in turn be interpreted as the marginal change in demand (and hence production) for a unit change in the optimal resetted price. Second, looking at (21) the effect of allowing for a positive trend inflation is evident: it alters dramatically the dynamics of inflation. With respect to the standard NKPC obtained log-linearizing the model around zero inflation steady state, (21) both changes the parameters values on the standard NKPC variables and enriches the dynamic structure, adding more forward looking terms. Ascari (2004) analyses thoroughly the implications of this fact for the dynamics of a standard sticky price model. Third, comparing (21) and (23) it is clear the effect of allowing for LRI. As we noted, (23) and (24) are the same as (21) and (22) respectively, where simply $\bar{\pi}$ is replaced by $\bar{\pi}^{(1-\varepsilon)}$. The effect of allowing for positive trend inflation is therefore counterbalanced by the indexation parameter. Hence these two effects, positive long-run inflation and indexation, go in opposite directions completely offsetting each other when indexation is full. Finally, allowing for PI adds another feature to the NKPC: it alters its dynamic structure even further, since it produces a change in the dynamics of both $\hat{\pi}_t$ and $\hat{\psi}_t$. As we know from Christiano, Eichenbaum, and Evans (2005), in this case lagged inflation enters the NKPC, generating some inflation inertia. Besides, current inflation enters the dynamic equation for $\hat{\psi}_t$ that instead was previously just depending on future inflation.

3.2 The inefficiency loss

There is however another very important effect that comes into the model when this latter is generalized to positive trend inflation. To see this, note that at the firm level it is true that

$$Y_t(i) = N_t(i) = \left[\frac{p_t(i)}{P_t}\right]^{-\theta} Y_t \tag{27}$$

but at the aggregate level there is no exact correspondence between Y_t and N_t. Indeed integrating the previous equation over i yields

$$N_t = \int_0^1 N_t(i)\,di = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t di = Y_t \underbrace{\int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta} di}_{s_t}.$$

So, the relation between aggregate output and aggregate labour is given by

$$Y_t = \frac{N_t}{s_t}$$

Schmitt-Grohe and Uribe (2004b) shows that s_t is bounded below one, so that s_t represents the resource costs due to relative price dispersion under the Calvo mechanism with long-run inflation. Indeed, the higher s_t, the more labour is needed to produce a given level of output. Note that s_t can also be rewritten as a ratio between two different price indexes P_t and $\widetilde{P}_t$

$$s_t = \left(\frac{P_t}{\widetilde{P}_t}\right)^{\theta} \text{ where } \widetilde{P}_t = \left[\int_0^1 P_t(i)^{-\theta}\,di\right]^{-1/\theta}$$

as in Yun (1996) and Ascari (2004). Whenever there is price dispersion these two indexes evolves differently from each other, determining a certain dynamics for s_t, that negatively affects the level of production. As Schmitt-Grohe and Uribe (2004a,b) noted, s_t would not affect the real variables up to first order whenever there is no trend inflation (i.e., $\overline{\pi} = 1$) or whenever the resetted price is fully indexed to any variable whose steady state level grows at the rate $\overline{\pi}$.[9]

For the purpose of this paper, it is important to stress that allowing for positive trend inflation and partial indexation makes a new variable to come into the model, i.e., s_t, that determines: (i) an inefficiency loss in aggregate production due to price dispersion; (ii) a further change in the dynamics of the model, as we see next.

[9] Indeed, if the fixed price is partially indexed to $\overline{\pi}$, P_t and $\widetilde{P}_t$ evolve respectively up to first order according to

$$\hat{P}_t = \alpha\overline{\pi}^{(\theta-1)(1-\varepsilon)}\widehat{P}_{t-1} + \left(1-\alpha\overline{\pi}^{(\theta-1)(1-\varepsilon)}\right)\hat{p}^*_{it}$$
$$\widehat{\widetilde{P}}_t = \alpha\overline{\pi}^{\theta(1-\varepsilon)}\widehat{\widetilde{P}}_{t-1} + \left(1-\alpha\overline{\pi}^{\theta(1-\varepsilon)}\right)\hat{p}^*_{it}.$$

Hence it is evident that if either $\overline{\pi} = 1$ (i.e., no trend inflation) or $\varepsilon = 1$ (full indexation), up to first order the dynamics of the two price indexes are the same. Schmitt-Grohe and Uribe (2004b) also stresses that this is not the case up to second order.

3.2.1 Dynamics of s_t

We show in the Appendix that s_t has the following backward looking dynamics.

1. No indexation

If there is no indexation then s_t evolves according to the following law of motion

$$s_t = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\pi_t^{\theta} s_{t-1} \tag{28}$$

that the Appendix shows it can be log-linearized to

$$\hat{s}_t = \frac{\theta}{\bar{\Omega}}\left[\bar{\pi}-1\right]\hat{\pi}_t + \alpha\bar{\pi}^{\theta}\hat{s}_{t-1} \tag{29}$$

where $\bar{\Omega} = \frac{1-\alpha\bar{\pi}^{\theta-1}}{\alpha\bar{\pi}^{\theta-1}}$.

2. LRI

If instead prices are partially indexed to long-run inflation, then s_t evolves as

$$s_t = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\left(\frac{\pi_t}{\bar{\pi}^{\varepsilon}}\right)^{\theta} s_{t-1} \tag{30}$$

that the Appendix shows it can be log-linearized to

$$\hat{s}_t = \frac{\theta}{\tilde{\Omega}}\left[\bar{\pi}^{1-\varepsilon}-1\right]\hat{\pi}_t + \alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1} \tag{31}$$

where $\tilde{\Omega} = \frac{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}$.

3. PI

Similarly, if prices are partially indexed to past inflation, the dynamics of s_t is described by

$$s_t = (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\varepsilon}}\right)^{\theta} s_{t-1} \tag{32}$$

that the Appendix shows it can be log-linearized to

$$\hat{s}_t = \frac{\theta}{\tilde{\Omega}}\left[\bar{\pi}^{1-\varepsilon}-1\right](\hat{\pi}_t - \varepsilon\hat{\pi}_{t-1}) + \alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1} \tag{33}$$

where $\tilde{\Omega} = \frac{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}$.

The important thing to note is that $\hat{s}_t$ has a backward looking dynamics and such further changes (and complicate) the dynamic structure of our model economy.

Finally, we just need an expression for the real marginal cost, which in the present case is common to all firms and simply equals to the real wage. Using the first order condition for consumption/labour choice, i.e., (19), the resource constraint $Y_t = C_t$ and $N_t = s_t Y_t$ yields

$$MC_t = \frac{W_t}{P_t} = \chi \frac{N_t^{\sigma_n}}{C_t^{-\sigma_c}} = \chi s_t^{\sigma_n} Y_t^{\sigma_c+\sigma_n}. \tag{34}$$

We then log-linearize it to obtain

$$\sigma_n \hat{s}_t + (\sigma_c + \sigma_n)\hat{Y}_t = \hat{mc}_t. \tag{35}$$

According to the assumed indexation scheme, we have three model economies. Each of them is described by five log-linearised equations. (20) and (35) are common to all models. The other three equations, instead, regards the dynamics of $\hat{\pi}_t$, $\hat{\psi}_t$ and $\hat{s}_t$, and thus they depend on the indexation scheme: (i) (21), (22) and (29) in the no indexation case; (ii) (23), (24) and (31) in the LRI case; (iii) (25), (26) and (33) in the PI case. The endogenous variables are: $\hat{\pi}_t, \hat{Y}_t, \hat{mc}_t, \hat{\psi}_t, \hat{s}_t$, plus the instrument of monetary policy $\hat{\imath}_t$. To close the model and to endogenize the policy instrument we will consider the most commonly employed Taylor-type rules in the literature.

Note that with respect to the standard case that assumes $\bar{\pi} = 1$, allowing for positive trend inflation and partial indexation makes the model economy more realistic, and also much richer both in terms of convolution of parameters and, above all, in terms of dynamic structure. In particular, the model has now two more variables that are absent in the standard case. They significantly alters the dynamic structure of our model economy: $\hat{\psi}_t$ is a forward-looking variable, while $\hat{s}_t$ is a backward-looking one. It is therefore not surprising that the dynamic properties of these models under Taylor-rule policies could be quite different from the standard case. This is what we move next.[10]

4 Contemporaneous Rule

The first Taylor-type rule we analyze is the classic contemporaneous monetary policy rule that, as in Taylor (1993, 1999), portrays the central bank as setting the nominal

[10] Moreover, note that substituting the dynamic equation for $\hat{s}_t$ into (35), we can obtain a dynamic equation for the marginal cost that is just function of the aggregate variables. Above all, this equation implies a backward-looking dynamics, that is a persistent behaviour of the marginal cost. The implication of this is the subject of ongoing research.

interest rate as function of current inflation and output gap, that is

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t. \tag{36}$$

Moreover, we set $\alpha = 0.75$, $\beta = 0.99$, $\theta = 11$, $\sigma_c = 1$ and $\sigma_n = 1$.

In what follows we will first consider the effects of trend inflation on determinacy in the no indexation case. Second, we then will generalize these results to the cases of partial (and full) indexation. Third, we will consider inertial Taylor rules. Fourth, we will present a dynamic analysis displaying the impulse response functions and analyzing how the dynamic response of the economy changes with trend inflation. Finally, we will consider the resulting unconditional variances of $\hat{\pi}_t$ and $\hat{Y}_t$.

4.1 Indeterminacy and the *Taylor principle*

Figure 1 depicts determinacy regions in the parameter space (ϕ_π, ϕ_Y) in the no indexation case for different levels of annualized trend inflation, 0%, 2%, 4%, 6% and 8%.[11] As stated in footnote 1, the average inflation in developed countries in post-war period fits in this range. A first result is visually very evident.

Result 1 Indeterminacy. *Positive levels of long run inflation greatly and unambiguously affect the determinacy properties of the rational expectation equilibrium. In particular, as $\bar{\pi}$ increases determinate regions fairly rapidly contract, ruling out implementable policy rules and increasing the likelihood of sunspots fluctuations.*[12]

Figure 1 well-renders how moderate levels of long-run inflation severely narrow the determinacy region, whose boundaries close like scissors. Besides, Table I gives a flavour about the considerable strength of this effect, by calculating the number of pairs (ϕ_π, ϕ_Y) that delivers determinacy given the step of our grid search in the simulations. Indeed, mildly increasing trend inflation from 0% to 2%, for example, produces a marked turn

[11] In Figure 1, as well as the following ones, we let $\phi_\pi \epsilon [0, 5]$ and $\phi_Y \epsilon [-1, 5]$. The grid search we use to discern determinate combinations of ϕ_π and ϕ_Y takes a step increase of 0.05. This means that in each run of our routine we check 12 221 different interest rate rule specifications.

[12] Note that instability, in the sense of explosive behavior, never arises in this region of the parameter space.

of determinacy frontier in Figure 1 with associated a cospicuous removal of determinate pairs which drop from 9232 down to 4293, approximately -53.5%. For economies featuring levels of trend inflation of 4% or 6%, the shrinking is by far more dramatic. Finally, at 8% trend inflation level, the only possible option for monetary policy to keep control of the economy is to strongly react to inflation and not to react to output gap. With respect to the zero trend inflation case the region shrinks by a striking 99%, meaning that only 1% of the pairs (ϕ_π, ϕ_Y) that lead to a determinate equilibrium in that case continue to deliver determinacy when $\bar{\pi} = 8\%$. In such a case, there is hardly any possible choice available to monetary policy to design an eventually optimal one.

Having shown trend inflation makes more likely indeterminacy, we now turn to discuss the implications from the point of view of the shape of determinacy regions. A second key result is stated in the following.

Result 2 The "Taylor principle". *Restrictions on policy coefficients valid under zero inflation steady state do not generalize to the case of positive trend inflation. In particular, the original "Taylor (1993, 1999) principle" (i.e., $\phi_\pi > 1$) breaks down. A generalized Taylor principle, however, still holds, but it is no longer a sufficient condition (in the positive orthant of the space ϕ_π, ϕ_Y).*

Taylor (1993, 1999) suggests that the monetary policy rule (36) should feature a value of ϕ_π bigger than one. In this case, the nominal interest rate rises by more than the increase of inflation, determining an increase in the real interest rate that will curb aggregate demand, thus guiding inflation expectations back to the rational expectation equilibrium. The literature then labelled the condition $\phi_\pi > 1$ as the *Taylor principle.*

If ϕ_π, $\phi_Y > 0$, however, it is well-known that in the standard microfounded Neo-Keynesian model featuring zero-inflation steady state a contemporaneous interest rate rule delivers a determinate rational expectations equilibrium if and only if (see, e.g., Bullard and Mitra, 2002, and Woodford, 2003, chp. 4)

$$\phi_\pi + \frac{(1-\beta)}{\kappa}\phi_Y > 1 \qquad (37)$$

where κ represents the elasticity of inflation to the output gap in the standard NKPC. As stressed by Bullard and Mitra (2002) and Woodford (2001, 2003) among others, condition (37) still corresponds to the *Taylor principle* in the sense that the nominal interest rate should rise by more than the increase of inflation in the long run. Indeed, as thoroughly discussed in Woodford (2003, chp. 4.2.2), $\frac{(1-\beta)}{\kappa}$ corresponds to the long run multiplier of the inflation rate on output in a standard NKPC log-linearized around the zero-inflation steady state. Hence the right-hand side of (37) "represents the long-run increase in the nominal interest rate prescribed [...] for each unit permanent increase in the inflation rate" (Woodford, 2003, p. 254). Therefore "The Taylor principle continues to be a crucial condition for determinacy, once understood to refer to *cumulative* responses to a *permanent* inflation increase" (Woodford, 2003, p. 256). As such, some authors identify the original Taylor principle (i.e., $\phi_\pi > 1$) with the more general condition (37).

Interestingly, we can generalize further more the discussion in Woodford (2003) to the trend inflation case. More generally, we can write (37) as

$$\frac{\partial \hat{\imath}}{\partial \hat{\pi}}|_{LR} = \phi_\pi + \phi_Y \frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR} > 1 \tag{38}$$

(where LR stands for long run). Appendix 7.5.1 calculates $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ in our model economy, to get "the long-run increase in the nominal interest rate prescribed [...] for each unit permanent increase in the inflation rate". $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ is a complicated expression (see equation (89)) that: (i) depends on trend inflation; (ii) for standard calibration values, it turns negative very soon as trend inflation is positive; (iii) for standard calibration values, it increases in absolute value as trend inflation increases. Moreover, plotting then (38) for different values of trend inflation, we exactly obtain the left-lateral frontier in Figure 1.

The Taylor principle therefore continues to be a crucial condition for determinacy in its general form (38), even in the case of trend inflation.[13] Trend inflation, however, significantly changes the implications of (38).

Indeed, in a zero inflation steady state, condition (38) reads as (37) and it is a necessary and sufficient condition in the positive orthant of the space (ϕ_π, ϕ_Y). Besides, the monetary policy literature amply focussed and discussed two main implications of restriction (37). First, it implies a trade off between ϕ_π and ϕ_Y: values of ϕ_π less than

[13] Again see the discussion in footnote 27, p. 256, in Woodford (2003).

one are admissible if the central bank appropriately compensate with relatively higher values of ϕ_Y, thus becoming more aggressive on output deviations. Second, actually this trade-off is very weak. Since β is usually calibrated to be very close to one in quantitative analysis, and since $\phi_\pi > 1$ is a sufficient condition for (37) to be satisfied, then the so-called Taylor principle is often practically referred to $\phi_\pi > 1$. Indeed, the literature mainly concentrated on ϕ_π, and so in what follows we will refer to the *Taylor principle* as $\phi_\pi > 1$. As a consequence, the value of ϕ_Y has always been considered as unimportant for determinacy.

As soon as one moves to non-zero steady state inflation economies, however, condition (38) ceases to be a sufficient condition in the positive orthant of the space (ϕ_π, ϕ_Y). The lower bound frontier, in fact, shifts upwards and it crosses the line defined by condition (38) in the positive orthant. More importantly, both the implications of condition (37) are rapidly and steadily turned upside down. First, even for moderate levels of $\bar{\pi}$ the aforementioned negative relation between ϕ_π and ϕ_Y on the left-lateral frontier immediately turns into positive, such that there is no trade-off between the two (while the lower-bound frontier turns counter clockwise). Indeed, along that frontier, if the central bank wants to lower ϕ_π it must at the same time respond less aggressively to the output gap to avoid indeterminacy. Equivalently, a central bank much concerned with output variations it has to be even tighter on inflation. Moreover, the higher trend inflation the flatter the left-lateral frontier and the larger the increase in ϕ_π per unit of ϕ_Y.

Second, restriction $\phi_\pi > 1$ is clearly shown to be not sufficient for determinacy, because the smallest determinate value of ϕ_π positively co-moves with $\bar{\pi}$. In the case of 6% inflation, for example, ϕ_π needs to be roughly higher than two. In addition, and more importantly, the coefficient on output gap now plays a key role, even for low values of trend inflation. As an example, in Figure 1 we evidentiate with a dot the point that corresponds to the canonical Taylor rule where $\phi_\pi = 1.5$ and $\phi_Y = 0.5$. As evident from the graph, as soon as trend inflation is bigger than 2% the Taylor rule creates indeterminacy. Hence, in real world application, the value of ϕ_Y cannot be neglected and it should basically be very low for realistic values of trend inflation.

To fully understand the results above, it is crucial to stress they basically depend on the long-run properties of the model and, particularly on the steady state relationship between inflation and output, a surprisingly neglected issue in the literature. The long-run Phillips curve is highly non-linear in the Neo-Keynesian model. As discussed in Ascari (1998) and Ascari (2004), it is positively sloped when $\bar{\pi} = 1$ (because of a discounting effect), but then turns quite rapidly negative even for extremely low value of trend inflation, because of the strong price-dispersion effect. As we show above, this has quite radical implications on the celebrated Taylor principle. The results in most of the literature are therefore based on a case (i.e., $\bar{\pi} = 1$) that is theoretically very special (as well as empirically unrealistic).

All in all, Figure 1 persuasively suggests that as trend inflation takes up higher values implementable monetary rules are characterized by an increasingly large and positive coefficient on inflation deviations and a very small, if not zero, coefficient on output gap. Essentially, this translates in the envision of a central bank that, as $\bar{\pi}$ increases, should increasingly be more concerned with inflation variations and eventually becomes a strict inflation targeting.

Furthermore, note that these results are in line with the analysis of Schmitt-Grohe and Uribe (2004a,b) and of Bullard and Mitra (2002). Even if dealing with two very different problems, both these papers robustly suggest monetary policy rule characterized by a high coefficient on ϕ_π and a close to zero coefficient on ϕ_Y. Despite our analysis tackles still another issue, it does deliver the same policy prescription for central banks behavior. Indeed, we find that whenever $\bar{\pi}$ is allowed to be positive, the determinate region shrinks towards those values being the only admissible ones.

Finally, it follows that allowing for positive trend inflation puts into question the validity of the *leaning against the wind* optimal policy in Clarida et al. (1999). As trend inflation increases, central bank can not afford to respond to output, but is should just concentrate on inflation. Ascari and Ropele (2004) indeed shows this to be true also for the optimal policy and provides basic intuition of why this happens.

4.2 Indexation

In this section we look at how our results change if we allow non-adjusting firms to index their prices. As in the previous section, Figure 2 shows the determinacy regions in the space (ϕ_π, ϕ_Y) for different levels of annualized trend inflation, 0%, 2%, 4%, 6% and 8% allowing for partial indexation ($\varepsilon = 0.5$) and full indexation in the two cases described in Section 2.3.

Result 3 *Allowing for indexation counteracts the effects of trend inflation described in the previous Section.*

The qualitative results of the previous section are still valid, while not surprisingly partial indexation tends to mitigate the effect of $\bar{\pi}$ on the determinacy regions. As illustrated in panels (a) and (b) of Figure 2, when $\varepsilon^{LR} = \varepsilon^{P} = 50\%$, positive levels of trend inflation do still narrow down determinacy regions although the contraction is now more sluggish. Importantly, indexation makes the lowest possible value of ϕ_π much less sensitive to trend inflation. Indeed, in Figure 1 ϕ_π needs to be at least 2 or greater than 3 to guarantee determinacy in the cases of 6% and 8% trend inflation respectively. When partial indexation is allowed, instead, the smallest implementable value for ϕ_π moves only slightly from 1. Finally, and paralleling the case of no indexation, the overall picture shows once again that as trend inflation increases the central bank has a smaller set of available implementable policies that force the monetary authority to respond more to inflation deviations and less to output gap, implying no trade-off between ϕ_π and ϕ_Y.

Note that full indexation completely neutralize any effect of trend inflation, since it is clear from Section 2.3 that when $\varepsilon = 1$ trend inflation does not enter any structural equations. Therefore, whatever the value of $\bar{\pi}$, full indexation (both in the case of past and long-run indexation) makes the model behaves as in the case of zero trend inflation *and* full indexation.[14]

[14] Indeed, the reader should be careful here. By looking at the formulas in Section 3.1, it is easy to check that: (i) in the case of indexation to long-run inflation, the structural equations under *zero inflation and no indexation* coincides with the ones under full indexation; (ii) this is instead not true for the case of indexation to past inflation. In this latter case, in fact, indexation changes the dynamics of the structural equations (see (25)) and hence the dynamic properties of the model economy. If indexation if full, then, trend inflation does not matter for the dynamic properties of the model economy (since the

Result 4 *For a given level of trend inflation, indexation to past inflation always delivers a set of implementable policies greater that the case of long-run indexation. Moreover, full indexation to past inflation restores the original Taylor principle ($\phi_\pi > 1$) a necessary and sufficient condition for determinacy.*

A final point we make regards the differences that characterize the two indexation schemes. Table 1 reports the number of implementable rules with partial indexation. For any level of trend inflation, partial PI is less likely to deliver sunspots fluctuations than the case of LRI. Moreover, even under long-run price stability (i.e., $\bar{\pi} = 0$), full PI exhibits a bigger set of implementable policies with respect to the other two cases (i.e., no indexation and LRI, which are indistinguishable when $\bar{\pi} = 0$).

Having said that, it is worth observing that the set of implementable policies in the case of PI is enlarged with respect to the case of LRI mainly because the lower bound frontier tilts downward, while the left-lateral frontier exhibits a very similar behavior in the two cases. This means that most of the extra policy options available for the monetary authority in the PI case regards the peculiar possibility of more pro-cyclical monetary policy (i.e., more negative values of ϕ_Y). In the other words, the central bank can still ensures determinacy of equilibria if remains looser on inflation deviations but, oddly enough, respond more pro-cylically to $\hat{Y}_t$.

Moreover, full PI restores the pivotal role of the original Taylor principle, i.e., $\phi_\pi > 1$. Indeed, quite interestingly, with 100% PI, $\phi_\pi > 1$ becomes a necessary and sufficient condition for determinacy. PI acts on the lower bound frontier which moderately turns clockwise and eventually becomes vertical when indexation is full in correspondence of $\phi_\pi = 1$[15].

4.3 Inertial policies

Empirical works on Taylor rules show that central banks tend to gradually adjust the nominal interest rate in response to changes in economic conditions (see, e.g., Rudebusch,

supply side equations do not depend on $\bar{\pi}$), but indexation does obviously matter. This explains why the model economy under *zero inflation and no indexation* behaves differently from the one characterized by *full indexation to past inflation (whatever trend inflation).*

[15] This means that also the left-lateral frontier moves with the indexation parameter, for any given level of trend inflation and becomes vertical whit 100% PII at $\phi_\pi = 1$.

1995, Judd and Rudebusch,1998 or Clarida et al., 2000). Moreover recent literature has emphasized the importance of inertial central bank behavior in the conduct of monetary policy with a forward-looking private sector (e.g., Woodford, 2003). Thus, in this section we explore the effects of positive trend inflation on the determinacy properties when our contemporaneous Taylor rule is modified to be

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{\imath}_{t-1}. \tag{39}$$

Figure 3 reports our numerical results displaying four panels where $\phi_i = 0.5, 1, 2$ and 5. Each panel represents determinacy regions in the parameter space (ϕ_π, ϕ_Y) for different values of $\bar{\pi}$, holding the remaining parameters at their baseline values.

Result 5 *Interest rate inertia makes indeterminacy less likely.*

The top-left panel of Figure 3 illustrates the case of $\phi_i = 0.5$. Compared to Figure 1 where $\phi_i = 0$, we note that a positive degree of monetary policy inertia visibly enlarges the determinacy region of the parameter space. The benefits of policy inertia become increasingly more pronounced, as the central bank controls the rate of change of nominal interest rate, rather than its level, as shown in panel (b), and as we consider, in the terminology of Woodford (2003) *explosive* or *superinertial* monetary policy rules, that is $\phi_i = 2$ and $\phi_i = 5$ (see panels (c) and (d)).

The somewhat counterintuitive feature that explosive rules enlarge the determinacy region therefore survives in the trend inflation case. As discussed in Rotemberg and Woodford (1999), in a similar model but with zero inflation steady state, it is exactly the possibility of explosiveness of the nominal interest rate that keeps the model on track.[16]

Result 6 *Interest rate inertia makes the Taylor principle (i.e., $\phi_\pi > 1$) plainly insignificant, in the sense that it is the value of ϕ_Y that actually matters for determinate equilibria. Surprisingly, monetary policy should not respond too much to output gap to prevent indeterminate equilibria.*

[16] Needless to say that, of course, the case of no feedback from inflation and output gap on the nominal interest rate (i.e., $\phi_\pi = \phi_Y = 0$) is of course indeterminate, for values of ϕ_i bigger than 1.

More significant is however the combined effect that inertial policy *and* trend inflation has on the validity of the Taylor principle. In the standard model with zero steady state inflation, Woodford (2003) shows that condition (37) becomes

$$\phi_\pi + \frac{(1-\beta)}{\kappa}\phi_Y > 1 - \phi_i \tag{40}$$

and therefore inertia enlarges the determinacy region, such that $\phi_i = 1$ is a sufficient condition for a determinate equilibrium. Moreover, a sufficient condition $\phi_\pi > 1 - \phi_i$ can be easily checked from any Taylor rule estimate. Note that this latter implies no role for ϕ_Y.

Again trend inflation radically changes the implications of the model. Indeed, looking at panel (b), it is evident that there is no more necessary condition on ϕ_π (provided that is positive); on the contrary, we can eventually state a sufficient condition on ϕ_Y. When $\bar{\pi}$ equals 4% or 8%, a sufficient condition for determinacy is ϕ_Y lower than 2 or 0, respectively, whatever the (positive) value of ϕ_π. In particular, for moderate levels of trend inflation (4% to 8%) what matters for determinacy is that monetary policy should basically not respond to the output gap.

As stressed in Section 4.1, this is due to the change in the sign of $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$. The relevant frontier is then positively sloped in the space (ϕ_π, ϕ_Y), such that monetary policy can afford to be looser on inflation *only if* it simultaneously becomes looser on output. In other words, being tough on output gap can easily generate indeterminacy, when monetary policy is characterized by an inertial (or superinertial) Taylor rule and moderate trend inflation.

Finally, it is easy to interpret graphically the effect of inertia in setting the interest rate, by comparing Figure 1 and the panels in Figure 3. The left-lateral frontier still obeys to a generalized Taylor principle of the form[17]

$$\phi_\pi + \phi_Y \frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR} > 1 - \phi_i \tag{41}$$

while the lower bound frontier exits the positive orthant. As a consequence, the crossing point of these two frontiers moves leftward as inertia increases. This in turn determines our results above: (i) the enlargement of the determinacy region; (ii) the consequent

[17] Indeed note that in panel (b) the lines pass through the point $(\phi_\pi = 0, \phi_Y = 0)$.

irrelevance of the original Taylor principle (i.e., $\phi_\pi > 1$); (iii) the crucial role of ϕ_Y that multiplies a negative quantity.

4.4 Dynamic Analysis

4.4.1 Impulse response functions

We now conduct some dynamic simulations and address the question of how trend inflation and indexation affect the dynamic properties of the model economy, in terms of impulse response functions and output/inflation variance frontier.[18]

To this purpose, as in Galí (2003), first we add to the equation of $\hat{\pi}_t$ a cost-push shock u_t, whose law of motion is

$$u_t = \rho u_{t-1} + \eta_t$$

where $0 \leq \rho < 1$ and η_t is a i.i.d. random variable with zero mean and variance σ_η^2. Second, we need to choose specific values for $\phi_\pi$and ϕ_Y, and we stick to the original Taylor specification, setting $\phi_\pi = 1.5$ and $\phi_Y = 0.5$ (and $\phi_i = 0$).[19]

Figure 4 displays the impulse response functions of output gap, inflation rate, nominal and real interest rate to a 1% cost-push shock, setting $\rho = 0.8$, both in the case of no indexation (the left column) and PI (the right column). Each panel reports different time patterns associated to increasing levels of trend inflation, for which the model is locally determinate.

The top-left panel displays the standard case of zero inflation steady state and no indexation. In response to a unit cost-push shock the monetary feedback rule calls for a sufficiently large increase in the nominal interest so to generate a positive real interest rate. Such a response, in turn, opens up a series of negative output gaps that gradually drives inflation rate back to neutral.

Result 7 *Increasing levels of positive trend inflation shift outward the impulse response functions of output and inflation, following a cost-push shock.*

[18] To solve for the rational expectation equilibrium and compute the impulse response function and variance frontiers we used the MATLAB version of Soderlind's codes available on the web.

[19] The results of this section do not qualitatively change if other values of $\phi_\pi$and ϕ_Y are chosen.

Consider now the case of 2% long-run inflation and no indexation.[20] Although the qualitative patterns are very similar to the case of zero inflation steady state, some important differences emerge. First, a positive level of trend inflation visibly alters the impact effects by producing an outward shift. Second, the outward shift of the impulse response function remains effective throughout the whole return path to steady state thus suggesting a tighter monetary policy and a deeper recession. In short, consistently with the results in Ascari and Ropele (2004), the higher is trend inflation, the worse the trade-off monetary policy is facing: the deeper is the recession and the higher the deviation of inflation from its steady state level.

The second column of Figure 4 shows the effects of partial PI.[21] As before trend inflation, either 2% or 4%, shift outwards the impulse response functions both on impact and on the whole adjustment path. Note that however, the shape of the impulse response function is now much different. As stressed by Christiano et al. (2005), PI creates the hump-shape in the impulse responses of output and inflation, because of the relatively richer dynamic structure due to the inclusion of π_{t-1} in the New Keynesian Phillips curve. For the same reason, persistence also increases with respect to the model with no indexation. Moreover, Figure 4 shows the comparison with the full indexation case[22] (i.e., the thick solid line), whose impulse response is unaffected by the trend inflation level. In this case, we have π_{t-1} entering the NKPC with the highest value, while the variable s_t disappears from the model, such that the effect on the persistence of the impulse response is a priori ambiguous. Figure 4, however, visually shows that full indexation tends to increase persistence, other things equal.

4.4.2 Unconditional Variance Frontiers

We now turn our attention to analyse the effects of trend inflation and indexation on unconditional variances of output and inflation, the arguments that are typically thought

[20] For the standard Taylor rule, we can just plot two impulse responses, since, as shown in Figure 1, for trend inflation equals to 4% onwards the model with no indexation is indeterminate in this case.

[21] We do not report the impulse response functions for LRI because such indexation rule only generates a (downward) rescaling with respect to the no indexation case. Again, the effect of LRI is just to counterbalance the trend inflation one.

[22] As in Christiano et al. (2005).

to characterize the central bank's loss function.

Because there are two policy coefficients in our interest rate rule on which to construct the variability frontier we proceed as follows. For different levels of $\bar{\pi}$, we compute the loci of output-inflation variability points by varying, in turn, one of the argument in the subrange $[0, 3]$ and keeping the other fixed at a chosen value. When we vary ϕ_π, panels (a) and (c) of Figure 5, ϕ_Y is set to 0.5; while when we vary ϕ_Y, panels (b) and (d), ϕ_π is set to 2.5.[23]

Result 8 *Increasing levels of positive trend inflation shift outward the policy frontiers, leading to worse outcomes for both inflation and output variability.*

This is the main result of this section, and we think a quite important one. It is evidently shown by the strong outward shift of the variance frontiers in Figure 5.[24] The attainable points with zero trend inflation in the space (ϕ_Y, ϕ_π) are not anymore so as $\bar{\pi}$ rises: either an higher value of σ_Y^2 is necessary for the same σ_π^2 or viceversa. Moreover, as trend inflation increases, the policy frontier substantially shortens (i.e., it is characterized by a fewer number of points), because of the model entering the indeterminacy region.

Panels (c) and (d) clearly reveal that again LRI tends to offset the effects of trend inflation on the policy frontiers, neutralizing it in the case of full LRI.

Result 9 *For a given level of trend inflation $\bar{\pi}$, LRI shifts the locus of output-inflation variability frontier inwards, therefore offsetting the trend inflation effects.*

Moreover, as we know from previous sections, partial indexation makes determinacy regions larger, hence the loci plotted in panels (c) and (d) comprise more points.

Figure 6 depicts the case of PI. Here, the results are for certain aspects more surprising. Although, we still observe a substantial shift of variability loci towards a welfare-worsening territory as $\bar{\pi}$ increases, PI generates a much higher output and inflation

[23] This value is different from the standard Taylor rule one (i.e., $\phi_\pi = 1.5$) we used in the previous section, just for convenience of presentation. The forntiers would exhibit otherwise very few points as trend inflation increases, because the model would quickly enter the indeterminacy region.

[24] As shown in Ascari and Ropele (2005) the deterioration of output/inflation policy frontier as trend inflation incresaes is also present when the moneteary policy is conducted optimally under eihter discretion or commitment.

variability with respect to the case of no indexation or LRI. This is just the other side of the coin of the fact that this type of indexation increase the endogenous persistence of the model. This result suggests that actually PI may indeed match some empirical regularities as stressed by Christiano et al. (2005), but theoretically may be very difficult to justify because of imposing high costs on agents' welfare. Finally, it is important to note that full indexation does not deliver the lower frontier. In other words, partial indexation (e.g., 50% in the figure) yields an efficient frontier always below the one obtained in the case of full indexation for values of trend inflation up to 6%. This suggests that full PI would not be *optimal* in a new keynesian model.[25]

Result 10 *PI strongly worsens the policy frontier, for any given level of trend inflation. Moreover, full indexation does not deliver the lowest possible policy frontier.*

5 Robustness

5.1 Alternative Interest Rate Rules

In this section we explore whether the results of previous sections are robust to simple variants of the Taylor rule largely proposed in the literature. In particular we consider: forward-looking interest rate rule (FL, henceforth): $i_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_Y E_t \hat{Y}_{t+1} + \phi_i \hat{\imath}_{t-1}$; backward-looking interest rate rule (BL, henceforth): $\hat{\imath}_t = \phi_\pi \hat{\pi}_{t-1} + \phi_Y \hat{Y}_{t-1} + \phi_i \hat{\imath}_{t-1}$; and two types of hybrid interest rate rules: $\hat{\imath}_t = \phi_\pi E_t \hat{\pi}_{t+1} + \phi_Y \hat{Y}_t + \phi_i \hat{\imath}_{t-1}$ (H1, henceforth) and $\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y E_t \hat{Y}_{t+1} + \phi_i \hat{\imath}_{t-1}$ (H2, henceforth).

The general conclusion of this section is that the key results found in previous analysis extend to all these cases. In particular, trend inflation again substantially changes the determinacy region in the parameter space and the dynamic properties of the model economy.

Moreover, for the cases of FL, H1 and H2 rules, Figures 7 and 8 show indeed how increasing levels of trend inflation impact the determinacy regions, basically in the same way described in the previous sections. The upshot is once again a substantial reduction in the number of implementable interest rate rules, which according to Tables 3, 5 and 6 appear to be more severe in the case of H1 rule. This reduction is still mainly due to the

[25] This analysis is the subject of ongoing research.

clockwise movement, as trend inflation increases, of the equivalent of condition (37) for the different analyzed rules. Again, the Figures and Tables show that both indexation schemes and inertia have the same effects as above.

5.1.1 Lagged Interest Rate Rule

The case of a central bank following a lagged interest rate rule is somewhat more involved and deserves a separate comment. As already known in the literature (e.g., Bullard and Mitra, 2002), in contrast to all the cases discussed so far, the BL rule can generate explosiveness of the solution, such that the rational expectations equilibrium is unstable and, if perturbed, it never returns to the steady state. Look at panel (a) of Figure 9 which depicts the standard case of zero inflation steady state.[26] The panel is divided into four regions by two lines, one of which is almost horizontal at $\phi_Y = 2$, while the other corresponds to the equivalent of condition (38) in the BL case. There is, however, an important difference with respect to the previous cases: in the parameters space above the almost horizontal line at $\phi_Y = 2$, the determinacy region now lies *on the left of condition (37) and not on its right*, where instead the region is in this case explosive.[27]

The other panels of Figure 9 shows the effect of increasing trend inflation. Graphically it is still the same, because the line corresponding to (38) again visibly rotates clockwise.[28] However, due to the fact that now the determinacy region lies partly on the left and partly on the right of this line, the effect of trend inflation is less clear-cut. Roughly speaking, dividing the parameters space in two regions, as trend inflation increases: (i) above the almost horizontal line at $\phi_Y = 2$, the instability region progressively shrinks and gives way to new determinate combinations; (ii) below the almost horizontal line at $\phi_Y = 2$, the indeterminacy region enlarges and reduces the number of implementable (i.e., determinate) rules. Note that while (ii) is the usual effect analyzed

[26] This Figure is construcetd differently from the others, since we need to distinguish among three different regions for each panel: (i) determinacy = white region; (ii) indeterminacy = small dots region; (iii) instability = darkest region.

[27] Note that in the parameters space below the almost horizontal line at $\phi_Y = 2$, the determinacy region lies, instead, as usual below condition (37).

[28] The other almost horizontal line discriminating among the different regions in the parameter space is instead only little sensitive to changes in trend inflation for our calibration values. Basically, as trend inflation increases, it tends to become horizontal at $\phi_Y = 2$, moving counterclockwise.

in the previous sections, (i) is the peculiarity of the BL rule. Everything then rests on the relative strength and dynamic adjustment of (i) and (ii).

Looking at the size of the effects (i) and (ii), (i) is stronger such that, positive trend inflation delivers always a larger determinacy region with respect to the case of zero inflation steady state (see first panel of Table 4). From a pure point of view of implentability, in the BL case a positive level of long run inflation might hence be desiderable and convenient, as it considerably enlarges the set of determinate monetary rules.

Looking at the dynamic adjustment as $\bar{\pi}$ increases, then, (i) is quicker than (ii), so that initially the determinate pairs $(\phi_Y; \phi_\pi)$ increases. Soon the movement in (i) clears the whole upward region from explosive behavior, and only (ii) remains, such that further increasing trend inflation reduces the determinacy region, as reported in Table 4.

As trend inflation takes up higher values, then, a central bank following a lagged interest rate rule is progressively left with two options to guarantee determinacy. On the one hand, it might respond to inflation deviations and be more cautious towards output gap, in line with previous analysis. On the other hand, the central bank can instead respond aggressively to output gap, i.e. $\phi_Y > 2$, regardless to the value of ϕ_π. Again, trend inflation makes the Taylor principle useless and the value of ϕ_Y what matters most. In sum,

Result 11 *When the monetary authority sets the nominal interest rate as function of lagged inflation and lagged output gap (with no inertia), positive levels of long run inflation actually increase the set of determinate policy rules, relative to the case of zero inflation steady state.*

Moreover, as trend inflation increases: (i) the indeterminacy region expands; (ii) the explosive region decreases; (iii) the determinacy region initially enlarges and then reduces.

Regarding the effects of indexing prices or inclusion of an inertial term in the rule the general results discussed in the previous sections simply carry on also in this case. Besides, inertia has an additional effect: it shifts upward the almost horizontal line in

Figure 9. As a result, the effect described in (i) becomes progressively less important and disappear from the parameters space for superinertial policies. Being (i) the peculiarity of the BL rule, it follows that for highly inertial policies, the properties of the model economy under BL rules are very similar to the other monetary policy rules.

5.2 Sensitivity Analysis

In this section we analyze the robustness of our findings to a number of variations in our model structural parametrization for the case of contemporaneous rule.[29] In particular, Figure 10 shows the determinacy region changing in turn the following parameters values: $\theta = 4$, $\alpha = 0.5$, $\sigma_c = 5$and $\sigma_n = 5$.

As expected, a lower value of the elasticity of substitution across goods or a lower value of the Calvo parameter make the determinacy frontier to close less rapidly when compared to the baseline calibration (see panels (a) and (b) of Figure 10). This leaves room to a relatively larger set of implementable policies for a given value of trend inflation and, in principle, determinate rules are also possible for even higher values.[30] Moreover, the original Taylor specification turns out to be determinate for trend inflation levels up to 6%, in the case of $\theta = 4$, or also 8%, in the case $\alpha = 0.5$.

Considering higher values for the inverse of the intertemporal elasticity of consumption and labour supply, the results are again qualitatively identical to the one presented above (see panels (c) and (d) of Figure 10). The lower bound frontier is slightly more sensitive to changes in σ_c, since as trend inflation increases the upward shift becomes more pronounced. As a consequence the lower boundary cuts the left-lateral frontier in correspondence of higher values of ϕ_π. The smallest admissible values of ϕ_π is evidently pushed rightward, thus calling for an increasingly more conservative central bank.

The value of σ_c moreover turns out to be quite important for the FL, BL and H2 cases. As already noted by Bullard and Mitra (2002), Figure 11 displays that the position of the almost horizontal line that characterizes Figures 7, 8 and 9 is quite sensitive to the values of σ_c. Indeed, it shifts notably upwards with σ_c and this has strong effects on

[29] The qualitative effects of changes in the values these parameters are in accordance with intuition and robust across different type of rules, indexation and inertia.

[30] Low values of θ and α, in fact, imply higher values of sustainable inflation rate in the steady state. For $\theta = 4$ the upper bound on $\bar{\pi}$ rises to 34.6% (annually) and for $\alpha = 0.5$ rises to 29.1%.

the dimension of the determinacy/indeterminacy regions in our parameters space.

6 Conclusions

Despite average inflation in the post-war period in developed countries was moderately different from zero, most of the vast literature on monetary policy rules worked with models log-linearized around zero inflation.

In this paper, we generalize a standard Neo-Keynesian model with Calvo staggered price by taking a linear approximation around a general trend inflation level. Then we use a Taylor rule to describe monetary policy. We then look at how the properties of our model economy change as the trend inflation level varies.

The results show that trend inflation greatly affects the previous results in the literature. In particular moderate levels of trend inflation: (i) modify the determinacy region in the parameters space; (ii) alter the impulse response function of the model economy after a cost-push shock. In line with Ascari (2004) and Ascari and Ropele (2004), this paper therefore shows that the Neo-Keynesian framework is quite sensitive to variations in the trend inflation level, in the sense that higher trend inflation basically makes monetary policy much less effective in controlling the dynamics of the economy.

Here we mainly concentrated on the effects of trend inflation on the set of implementable monetary policy rules in order to deliver a determinate rational expectations equilibrium. We show that trend inflation substantially changes the celebrated determinacy condition that the literature labelled the *Taylor principle.*

Our key results are then generalized and proved to be robust to: (i) different kinds of Taylor type rules proposed in the literature; (ii) inertial Taylor rules for all the cases in (i); (iii) indexation schemes used in the literature; (iv) different parameter values.

In sum, this paper shows that the literature on monetary policy rules cannot neglect trend inflation both in the empirical and theoretical analysis, because all the results are very sensitive to low and moderate trend inflation levels, as empirically observed in western countries.

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7 Appendix

7.1 Derivation of the NKPC under no indexation

Here we provide details of the log-linearization of (11) which leads to the New Keynesian Phillips curve (21) and (22) in the main text. We begin by re-writing numerator and denominator of (11) as

$$\frac{p_t^*(i)}{P_t} = \frac{\theta}{\theta-1}\left(\frac{\psi_t}{\phi_t}\right)$$

where

$$\begin{aligned}\psi_t &= E_t \sum_{j=0}^{\infty} (\alpha\beta)^j u_c(t+j)\left[\left(\frac{P_{t+j}}{P_t}\right)^{\theta} Y_{t+j} MC_{t+j}(i)\right] \\ \phi_t &= E_t \sum_{j=0}^{\infty} (\alpha\beta)^j u_c(t+j)\left[\left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} Y_{t+j}\right].\end{aligned}$$

The denominator can also be written as:

$$\phi_t = u_c(t) Y_t + E_t \sum_{j=0}^{\infty} (\alpha\beta)^{j+1}\left[\left(\frac{P_{t+j+1}}{P_t}\right)^{\theta-1} u_c(t+j+1) Y_{t+j+1}\right].$$

Next, considering the definition for ϕ_{t+1} and collecting the term $\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1}$ yields the following expression for ϕ_t

$$\phi_t = u_c(t) Y_t + \alpha\beta E_t\left(\pi_{t+1}^{\theta-1}\phi_{t+1}\right) \tag{42}$$

where $\pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. Doing exactly the same steps for the numerator gives rise to the following expression for ψ_t

$$\psi_t = u_c(t) Y_t MC_t(i) + \alpha\beta E_t\left(\pi_{t+1}^{\theta}\psi_{t+1}\right). \tag{43}$$

Now we take a log-linear approximation of (42) and (43). ϕ is linearized around $\frac{u_c Y}{\left(1-\alpha\beta\bar{\pi}^{\theta-1}\right)}$, ψ around $\frac{u_c Y MC(i)}{1-\alpha\beta\bar{\pi}^{\theta}}$, Y_t around Y, π around $\bar{\pi}$ and $u_c(t)$ around $Y^{-\sigma_c}$

$$\begin{aligned}\hat{\psi}_t &\simeq \left(1-\alpha\beta\bar{\pi}^{\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{M}C_t\right] + \alpha\beta\bar{\pi}^{\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right] &(44)\\ \hat{\phi}_t &\simeq \left(1-\alpha\beta\bar{\pi}^{\theta-1}\right)\left[\hat{u}_c(t) + \hat{Y}_t\right] + \alpha\beta\bar{\pi}^{\theta-1}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right]. &(45)\end{aligned}$$

With this results at hand we can compactly rewrite the log-linearized optimal price (11) as

$$\hat{p}_t^*(i) - \hat{P}_t = \hat{\psi}_t - \hat{\phi}_t. \tag{46}$$

In order to find the New Keynesian Phillips curve we have to combine this last equation with the log-linear expression of the aggregate price level, which evolves according to

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta}\, di\right]^{\frac{1}{1-\theta}} = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha)P_t(i)^{1-\theta}\right]^{\frac{1}{1-\theta}} \Longrightarrow \tag{47}$$

$$1 = \left[\alpha \pi_t^{\theta-1} + (1-\alpha)\left(\frac{P_t(i)}{P_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{48}$$

Note that (48) implies that in steady state

$$\frac{p^*(i)}{P} = \left(\frac{1-\alpha\bar{\pi}^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}} \tag{49}$$

and hence the log-linearized version of (48) is

$$\hat{p}_{it}^* - \hat{P}_t = \left(\frac{\alpha\bar{\pi}^{\theta-1}}{1-\alpha\bar{\pi}^{\theta-1}}\right)\hat{\pi}_t. \tag{50}$$

Substituting (50) into (46), we obtain the New Keynesian Phillips curve under positive trend inflation and no indexation described by the following three equations

$$\hat{\pi}_t = \left[\frac{1-\alpha\bar{\pi}^{\theta-1}}{\alpha\bar{\pi}^{\theta-1}}\right]\left(\hat{\psi}_t - \hat{\phi}_t\right) \tag{51}$$

$$\hat{\psi}_t = \left(1-\alpha\beta\bar{\pi}^{\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{M}C_t\right] + \alpha\beta\bar{\pi}^{\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right] \tag{52}$$

$$\hat{\phi}_t = \left(1-\alpha\beta\bar{\pi}^{\theta-1}\right)\left[\hat{u}_c(t) + \hat{Y}_t\right] + \alpha\beta\bar{\pi}^{\theta-1}\left[(\theta-1)\,\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right]. \tag{53}$$

Interestingly enough the above system can be reduced to only two equations. First write the difference between $\hat{\psi}_t$ and $\hat{\phi}_t$ as

$$\begin{aligned}\left(\hat{\psi}_t - \hat{\phi}_t\right) = {} & \alpha\beta\bar{\pi}^{\theta-1}(1-\bar{\pi})(1-\sigma_c)\hat{Y}_t + \left(1-\alpha\beta\bar{\pi}^{\theta}\right)\hat{M}C_t + \\ & +\theta\alpha\beta\bar{\pi}^{\theta-1}(\bar{\pi}-1)\hat{\pi}_{t+1} + \alpha\beta\bar{\pi}^{\theta-1}\hat{\pi}_{t+1} + \alpha\beta\bar{\pi}^{\theta-1}\left(\bar{\pi}\hat{\psi}_{t+1} - \hat{\phi}_{t+1}\right)\end{aligned}$$

where we also used $\hat{u}_c(t) = -\sigma_c\hat{Y}_t$.

Next add and subtract $\alpha\beta\bar{\pi}^{\theta-1}\hat{\psi}_{t+1}$ so to have

$$\begin{aligned}\left(\hat{\psi}_t - \hat{\phi}_t\right) = {} & \alpha\beta\bar{\pi}^{\theta-1}(1-\bar{\pi})(1-\sigma_c)\hat{Y}_t + \left(1-\alpha\beta\bar{\pi}^{\theta}\right)\hat{M}C_t + \\ & +\theta\alpha\beta\bar{\pi}^{\theta-1}(\bar{\pi}-1)\hat{\pi}_{t+1} + \alpha\beta\bar{\pi}^{\theta-1}\hat{\pi}_{t+1} + \alpha\beta\bar{\pi}^{\theta-1}\left(\hat{\psi}_{t+1} - \hat{\phi}_{t+1}\right) \\ & +\alpha\beta\bar{\pi}^{\theta-1}(\bar{\pi}-1)\hat{\psi}_{t+1}.\end{aligned}$$

Plugging into the last expression $\left[\frac{\alpha\bar{\pi}^{\theta-1}}{1-\alpha\bar{\pi}^{\theta-1}}\right]\hat{\pi}_t = \left(\hat{\psi}_t - \hat{\phi}_t\right)$ yields to

$$\begin{aligned}\hat{\pi}_t &= \left(1-\alpha\bar{\pi}^{\theta-1}\right)\beta\left(1-\bar{\pi}\right)\left(1-\sigma_c\right)\hat{Y}_t + \frac{\left(1-\alpha\bar{\pi}^{\theta-1}\right)\left(1-\alpha\beta\bar{\pi}^{\theta}\right)}{\alpha\bar{\pi}^{\theta-1}}\hat{M}C_t + \\ &+ \left(1-\alpha\bar{\pi}^{\theta-1}\right)\theta\beta\left(\bar{\pi}-1\right)\hat{\pi}_{t+1} + \left(1-\alpha\bar{\pi}^{\theta-1}\right)\beta\hat{\pi}_{t+1} + \\ &+ \alpha\beta\bar{\pi}^{\theta-1}\hat{\pi}_{t+1} + \left(1-\alpha\bar{\pi}^{\theta-1}\right)\beta\left(\bar{\pi}-1\right)\hat{\psi}_{t+1}.\end{aligned}$$

Now using the definition $\hat{\psi}_t$ we can substitute for $\hat{\psi}_{t+1} = \frac{1}{\alpha\beta\bar{\pi}^{\theta}}\hat{\psi}_t - \frac{\left(1-\alpha\beta\bar{\pi}^{\theta}\right)}{\alpha\beta\bar{\pi}^{\theta}}\left[\hat{u}_c(t) + \hat{Y}_t + \hat{M}C_t\right] - \theta\hat{\pi}_{t+1}$, finally obtaining

$$\hat{\pi}_t = \lambda(\bar{\pi})\frac{\left(1-\bar{\pi}\right)\left(1-\sigma_c\right)}{\left(1-\alpha\beta\bar{\pi}^{\theta}\right)}\hat{Y}_t + \lambda(\bar{\pi})\hat{M}C_t + \beta\hat{\pi}_{t+1} + \lambda(\bar{\pi})\frac{\left(\bar{\pi}-1\right)}{\left(1-\alpha\beta\bar{\pi}^{\theta}\right)}\hat{\psi}_t \tag{54}$$

where $\lambda(\bar{\pi}) = \frac{\left(1-\alpha\bar{\pi}^{\theta-1}\right)\left(1-\alpha\beta\bar{\pi}^{\theta}\right).}{\alpha\bar{\pi}^{\theta}}$. (54) and (44) then fully describe the NKPC in the no indexation case and are reported in the main text.

7.2 Derivation of the NKPC under partial indexation to long-run inflation

Here we provide details of the log-linearization of (13) which leads to the New Keynesian Phillips curve (23) and (24) in the main text. We proceed along the same steps as above. Hence

$$\frac{p_t^*(i)}{P_t} = \frac{\theta}{\theta-1}\left(\frac{\psi_t}{\phi_t}\right) \qquad \Longrightarrow \qquad \hat{p}_t^*(i) - \hat{P}_t = \hat{\psi}_t - \hat{\phi}_t$$

where however in this case

$$\begin{aligned}\psi_t &= E_t\sum_{j=0}^{\infty}(\alpha\beta)^j u_c(t+j)\left[P_{t+j}^{\theta}Y_{t+j}MC_{t+j}^r(i)\,\Pi_{t,t+j-1}^{-\theta}\right] \\ \phi_t &= E_t\sum_{j=0}^{\infty}(\alpha\beta)^j u_c(t+j)\left[P_{t+j}^{\theta-1}Y_{t+j}\Pi_{t,t+j-1}^{(1-\theta)}\right].\end{aligned}$$

It is easy to check that

$$\begin{aligned}\hat{\psi}_t &= \left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{M}C_t\right] + \alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right] &(55)\\ \hat{\phi}_t &= \left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\right)\left[\hat{u}_c(t) + \hat{Y}_t\right] + \alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right]. &(56)\end{aligned}$$

The aggregate price level now evolves according to

$$\begin{aligned}P_t &= \left[\int_0^1 P_t(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}} = \left[\alpha\bar{\pi}^{(1-\theta)\varepsilon}P_{t-1}^{1-\theta} + (1-\alpha)P_t(i)^{1-\theta}\right]^{\frac{1}{1-\theta}} \Longrightarrow &(57)\\ 1 &= \left[\alpha\bar{\pi}^{(1-\theta)\varepsilon}\pi_t^{\theta-1} + (1-\alpha)\left(\frac{P_t(i)}{P_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. &(58)\end{aligned}$$

Note that (58) implies that in steady state

$$\frac{p^*(i)}{P} = \left(\frac{1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}{1-\alpha}\right)^{\frac{1}{1-\theta}} \tag{59}$$

and hence the log-linearized version of (58) is

$$\hat{p}^*_{it} - \hat{P}_t = \left(\frac{\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}{1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}\right)\hat{\pi}_t. \tag{60}$$

The New Keynesian Phillips curve under partial indexation to long-run inflation is therefore described by (55), (56), and

$$\hat{\pi}_t = \left(\frac{1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}{\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}\right)\left(\hat{\psi}_t - \hat{\phi}_t\right). \tag{61}$$

Note that these three equations are the same as in the previous case, where simply $\bar{\pi}$ is replaced by $\bar{\pi}^{(1-\varepsilon)}$. Therefore, proceeding as above the system can be reduced to only two equations, that is

$$\hat{\pi}_t = \lambda_{LR}(\bar{\pi})\frac{\left(1-\bar{\pi}^{(1-\varepsilon)}\right)(1-\sigma_c)}{\left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)}\hat{Y}_t + \lambda_{LR}(\bar{\pi})\hat{MC}_t + \beta\hat{\pi}_{t+1} + \lambda_{LR}(\bar{\pi})\frac{\left(\bar{\pi}^{(1-\varepsilon)}-1\right)}{\left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)}\hat{\psi}_t \tag{62}$$

where $\lambda_{LR}(\bar{\pi}) = \frac{\left(1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}\right)\left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)}{\alpha\bar{\pi}^{(1-\varepsilon)\theta}}$. (62) and (55) then fully describe the NKPC in the case of indexation to long run inflation, as reported in the main text.

7.3 Derivation of the NKPC under partial indexation to past inflation[31]

Here we provide details of the log-linearization of (15) which leads to the New Keynesian Phillips curve (25) and (26) in the main text. We proceed along the same steps as above. Hence

$$\frac{p^*_t(i)}{P_t} = \frac{\theta}{\theta-1}\left(\frac{\psi_t}{\phi_t}\right) \qquad \Longrightarrow \qquad \hat{p}^*_t(i) - \hat{P}_t = \hat{\psi}_t - \hat{\phi}_t$$

where however in this case

$$\begin{aligned}
\psi_t &= E_t\sum_{j=0}^{\infty}(\alpha\beta)^j u_c(t+j)\left[\left(\frac{P_{t+j}}{P_t}\right)^{\theta}\bar{\pi}^{-\theta\varepsilon j}Y_{t+j}MC_{t+j}(i)\right] \\
\phi_t &= E_t\sum_{j=0}^{\infty}(\alpha\beta)^j u_c(t+j)\left[\left(\frac{P_{t+j}}{P_t}\right)^{\theta-1}\bar{\pi}^{(1-\theta)\varepsilon j}Y_{t+j}\right].
\end{aligned}$$

[31] See also Maury and Sahuc (2004).

It is easy to check that

$$\begin{aligned}
\hat{\psi}_t &= \left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\right)\left[\hat{u}_c\left(t\right)+\hat{Y}_t+\hat{M}C_t\right]+\alpha\beta\bar{\pi}^{(1-\varepsilon)\theta}\left[\theta\hat{\pi}_{t+1}-\theta\varepsilon\hat{\pi}_t+\hat{\psi}_{t+1}\right] \qquad (63)\\
\hat{\phi}_t &= \left(1-\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\right)\left[\hat{u}_c\left(t\right)+\hat{Y}_t\right]+\\
&\quad +\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\left[\left(\theta-1\right)\hat{\pi}_{t+1}-\varepsilon(\theta-1)\hat{\pi}_t+\hat{\phi}_{t+1}\right]. \qquad (64)
\end{aligned}$$

The aggregate price level evolves according to

$$\begin{aligned}
P_t &= \left[\int_0^1 P_t\left(i\right)^{1-\theta}di\right]^{\frac{1}{1-\theta}}=\left[\alpha\pi_{t-1}^{(1-\theta)\varepsilon}P_{t-1}^{1-\theta}+(1-\alpha)P_t\left(i\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}\implies \qquad (65)\\
1 &= \left[\alpha\pi_{t-1}^{(1-\theta)\varepsilon}\pi_t^{\theta-1}+(1-\alpha)\left(\frac{P_t\left(i\right)}{P_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \qquad (66)
\end{aligned}$$

Note that (66) implies that in steady state $\frac{p^*(i)}{P}$ is the same as in the previous case, i.e., (59). The log-linearized version of (66) is thus

$$\hat{p}^*_{it}-\hat{P}_t=\left(\frac{\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}{1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}\right)(\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1}). \qquad (67)$$

The New Keynesian Phillips curve under partial indexation to long-run inflation is therefore described by (63), (64) and

$$\hat{\pi}_t=\varepsilon\hat{\pi}_{t-1}+\left(\frac{1-\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}{\alpha\bar{\pi}^{(1-\varepsilon)(\theta-1)}}\right)\left(\hat{\psi}_t-\hat{\phi}_t\right). \qquad (68)$$

Again we can follow the same steps as above to reduce the system to two equations. First write the difference between $\hat{\psi}_t$ and $\hat{\phi}_t$ as

$$\begin{aligned}
\hat{\psi}_t-\hat{\phi}_t &= \alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\left(1-\bar{\pi}^{(1-\varepsilon)}\right)\left(1-\sigma_c\right)\hat{Y}_t+\left(1-\alpha\beta\bar{\pi}^{\theta(1-\varepsilon)}\right)\hat{M}C_t+\\
&\quad +\theta\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\left(\bar{\pi}^{(1-\varepsilon)}-1\right)\hat{\pi}_{t+1}+\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\hat{\pi}_{t+1}+\\
&\quad -\theta\varepsilon\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\left(\bar{\pi}^{(1-\varepsilon)}-1\right)\hat{\pi}_t-\varepsilon\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\hat{\pi}_t+\\
&\quad +\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}\left(\hat{\psi}_{t+1}-\hat{\phi}_{t+1}\right)+\alpha\beta\bar{\pi}^{(1-\varepsilon)(\theta-1)}(\bar{\pi}^{(1-\varepsilon)}-1)\hat{\psi}_{t+1}
\end{aligned}$$

where we also used $\hat{u}_c\left(t\right)=-\sigma_c\hat{Y}_t$.

Using (68) to substitute for $\left(\hat{\psi}_t-\hat{\phi}_t\right)$ and $\left(\hat{\psi}_{t+1}-\hat{\phi}_{t+1}\right)$ into the last expression yields (25) in the main text.

7.4 The inefficiency loss s_t

7.4.1 Dynamics of s_t

Following Schmitt-Grohe and Uribe (2004b), p. 16, it is easy to derive the expressions for the dynamics of s_t in the main text.

1. No indexation

$$\begin{aligned} s_t &= \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta} di = \\ &= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha(1-\alpha)\left[\frac{P_{t-1}^*(i)}{P_t}\right]^{-\theta} + \alpha^2(1-\alpha)\left[\frac{P_{t-2}^*(i)}{P_t}\right]^{-\theta} + ... = \\ &= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\pi_t^{\theta}\left[(1-\alpha)\left[\frac{P_{t-1}^*(i)}{P_{t-1}}\right]^{-\theta} + \alpha(1-\alpha)\left[\frac{P_{t-2}^*(i)}{P_{t-1}}\right]^{-\theta} + ..\right] = \\ &= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\pi_t^{\theta} s_{t-1}. \end{aligned} \tag{69}$$

2. Partial indexation to trend inflation

$$\begin{aligned} s_t &= \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta} di = \\ &= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha(1-\alpha)\left[\frac{P_{t-1}^*(i)\,\pi^{\varepsilon}}{P_t}\right]^{-\theta} + \alpha^2(1-\alpha)\left[\frac{P_{t-2}^*(i)\,\pi^{2\varepsilon}}{P_t}\right]^{-\theta} + ... = \\ &= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\left(\frac{\pi_t}{\bar{\pi}^{\varepsilon}}\right)^{\theta}\left[(1-\alpha)\left[\frac{P_{t-1}^*(i)}{P_{t-1}}\right]^{-\theta} + \right. \\ &\quad \left. +\alpha(1-\alpha)\left[\frac{P_{t-2}^*(i)\,\pi^{\varepsilon}}{P_{t-1}}\right]^{-\theta} + ..\right] \\ &= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\left(\frac{\pi_t}{\bar{\pi}^{\varepsilon}}\right)^{\theta} s_{t-1}. \end{aligned} \tag{70}$$

3. Partial indexation to past inflation

$$
\begin{aligned}
s_t &= \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\theta} di = \\
&= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha(1-\alpha)\left[\frac{P_{t-1}^*(i)\,\pi_{t-1}^{\varepsilon}}{P_t}\right]^{-\theta} + \\
&\quad +\alpha^2(1-\alpha)\left[\frac{P_{t-2}^*(i)\,\pi_{t-1}^{\varepsilon}\pi_{t-2}^{\varepsilon}}{P_t}\right]^{-\theta} + \ldots \\
&= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\varepsilon}}\right)^{\theta}\left[(1-\alpha)\left[\frac{P_{t-1}^*(i)}{P_{t-1}}\right]^{-\theta} + \right. \\
&\quad \left. +\alpha(1-\alpha)\left[\frac{P_{t-2}^*(i)\,\pi_{t-2}^{\varepsilon}}{P_{t-1}}\right]^{-\theta} + ..\right] \\
&= (1-\alpha)\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^{\varepsilon}}\right)^{\theta} s_{t-1}.
\end{aligned} \tag{71}
$$

7.4.2 Steady states under different indexation schemes

1. No Indexation

In steady state,

$$\left[\frac{P_t^*(i)}{P_t}\right]_{SS} = \left[\frac{1-\alpha\bar{\pi}^{\theta-1}}{1-\alpha}\right]^{\frac{1}{1-\theta}} \tag{72}$$

hence from (69)

$$s = \frac{1-\alpha}{1-\alpha\bar{\pi}^{\theta}}\left[\frac{1-\alpha\bar{\pi}^{(\theta-1)}}{1-\alpha}\right]^{\frac{\theta}{\theta-1}}. \tag{73}$$

2. Indexation to long run inflation

From (70) we get

$$s = \frac{1-\alpha}{1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}}\left[\frac{P_t^*(i)}{P_t}\right]^{-\theta}$$

but under this indexation scheme in steady state

$$\left[\frac{P_t^*(i)}{P_t}\right]_{SS} = \left[\frac{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1-\alpha}\right]^{\frac{1}{1-\theta}} \tag{74}$$

thus

$$s = \frac{1-\alpha}{1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}}\left[\frac{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1-\alpha}\right]^{\frac{\theta}{\theta-1}}. \tag{75}$$

Note that this holds also for the case of **indexation to past inflation**. Indeed the steady state value of s does not depend on the type of indexation. Note that this holds generally whenever the resetted price is indexed to any variable that in steady state grows at the rate $\bar{\pi}$.

7.4.3 Log-linearization

For all the different indexation schemes, the following holds

$$\frac{P_t^*(i)}{P_t} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{\psi_t}{\phi_t}\right)$$

but one should be careful since each indexation scheme has an accordingly different definition of ψ_t and ϕ_t.

1. No indexation

From (69) and using (72) and (73), it yields

$$\begin{aligned} s_t &= (1-\alpha)\left[\left(\frac{\theta}{\theta-1}\right)^{-\theta}\phi_t^{\theta}\psi_t^{-\theta}\right] + \alpha\pi_t^{\theta}s_{t-1} \\ &\simeq \left[\frac{(1-\alpha)\left(\frac{\theta}{\theta-1}\right)^{-\theta}\theta\phi^{\theta}\psi^{-\theta}}{s}\right]\left(\hat{\phi}_t - \hat{\psi}_t\right) + \\ &\quad + \left[\frac{\alpha\bar{\pi}^{\theta}\theta s}{s}\right]\hat{\pi}_t + \left[\frac{\alpha\bar{\pi}^{\theta}s}{s}\right]\hat{s}_{t-1}. \end{aligned}$$

Thus

$$\hat{s}_t = \left(1-\alpha\bar{\pi}^{\theta}\right)\theta\left(\hat{\phi}_t - \hat{\psi}_t\right) + \alpha\bar{\pi}^{\theta}\left(\theta\hat{\pi}_t + \hat{s}_{t-1}\right). \tag{76}$$

We may want to substitute for $\left(\hat{\phi}_t - \hat{\psi}_t\right)$ to express $\hat{s}_t$ as a function of $\hat{\pi}_t$. Substituting then (51), we get

$$\begin{aligned} \hat{s}_t &= \left(1-\alpha\bar{\pi}^{\theta}\right)\theta\left(\hat{\phi}_t - \hat{\psi}_t\right) + \alpha\bar{\pi}^{\theta}\left(\theta\hat{\pi}_t + \hat{s}_{t-1}\right) = \\ &= \left(1-\alpha\bar{\pi}^{\theta}\right)\theta\left[\frac{-\alpha\bar{\pi}^{\theta-1}}{1-\alpha\bar{\pi}^{\theta-1}}\right]\hat{\pi}_t + \alpha\bar{\pi}^{\theta}\left(\theta\hat{\pi}_t + \hat{s}_{t-1}\right) = \\ &= \theta\left[\frac{-1+\alpha\bar{\pi}^{\theta}}{\bar{\Omega}} + \alpha\bar{\pi}^{\theta}\right]\hat{\pi}_t + \alpha\bar{\pi}^{\theta}\hat{s}_{t-1} \end{aligned}$$

where $\bar{\Omega} = \left(\frac{\left(1-\alpha\bar{\pi}^{(\theta-1)}\right)}{\alpha\bar{\pi}^{(\theta-1)}}\right)$. Hence, more compactly

$$\hat{s}_t = \frac{\theta}{\bar{\Omega}}\left[\alpha\bar{\pi}^{\theta}\left(1+\bar{\Omega}\right) - 1\right]\hat{\pi}_t + \alpha\bar{\pi}^{\theta}\hat{s}_{t-1}. \tag{77}$$

2. Indexation to long-run inflation

From (70) the law of motion of s_t becomes

$$\begin{aligned} s_t &= (1-\alpha)\left[\left(\frac{\theta}{\theta-1}\right)^{-\theta}\phi_t^{\theta}\psi_t^{-\theta}\right]+\alpha\left(\frac{\pi_t}{\bar{\pi}^{\varepsilon}}\right)^{\theta}s_{t-1} \\ &\simeq \left[\frac{(1-\alpha)\left(\frac{\theta}{\theta-1}\right)^{-\theta}\theta\phi^{\theta}\psi^{-\theta}}{s}\right]\left(\hat{\phi}_t-\hat{\psi}_t\right)+ \\ &\quad +\left[\frac{\alpha\bar{\pi}^{-\theta\varepsilon}\theta\bar{\pi}^{\theta}s}{s}\right]\hat{\pi}_t+\left[\frac{\alpha\bar{\pi}^{-\theta\varepsilon}\bar{\pi}^{\theta}s}{s}\right]\hat{s}_{t-1}. \end{aligned}$$

Using (75) and (74), it yields

$$\hat{s}_t=\left(1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right)\theta\left(\hat{\phi}_t-\hat{\psi}_t\right)+\alpha\bar{\pi}^{\theta(1-\varepsilon)}[\theta\hat{\pi}_t+\hat{s}_{t-1}]. \tag{78}$$

Finally, substitute (61), we obtain

$$\begin{aligned} \hat{s}_t &= \left(1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right)\theta\left(\hat{\phi}_t-\hat{\psi}_t\right)+\alpha\bar{\pi}^{\theta(1-\varepsilon)}[\theta\hat{\pi}_t+\hat{s}_{t-1}]= \\ &= \left(1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right)\theta\left[\frac{-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}\right]\hat{\pi}_t+\alpha\bar{\pi}^{\theta(1-\varepsilon)}[\theta\hat{\pi}_t+\hat{s}_{t-1}]= \\ &= \theta\left[\frac{-1+\alpha\bar{\pi}^{\theta(1-\varepsilon)}}{\tilde{\Omega}}+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right]\hat{\pi}_t+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1}= \\ &= \frac{\theta}{\tilde{\Omega}}\left[\alpha\bar{\pi}^{\theta(1-\varepsilon)}\left(1+\tilde{\Omega}\right)-1\right]\hat{\pi}_t+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1} \end{aligned}$$

where $\tilde{\Omega}=\left(\frac{\left(1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}\right)}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}\right)$. Hence

$$\hat{s}_t=\frac{\theta}{\tilde{\Omega}}\left[\alpha\bar{\pi}^{\theta(1-\varepsilon)}\left(1+\tilde{\Omega}\right)-1\right]\hat{\pi}_t+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1}. \tag{79}$$

3. Indexation to past inflation

The law of motion of s_t in this case is approximated to

$$\begin{aligned} s_t &= (1-\alpha)\left[\left(\frac{\theta}{\theta-1}\right)^{-\theta}\phi_t^{\theta}\psi_t^{-\theta}\right]+\alpha\left(\frac{\pi_t}{\pi_{t-1}^{\varepsilon}}\right)^{\theta}s_{t-1} \\ &= \left[\frac{(1-\alpha)\left(\frac{\theta}{\theta-1}\right)^{-\theta}\theta\phi^{\theta}\psi^{-\theta}}{s}\right]\left(\hat{\phi}_t-\hat{\psi}_t\right)+ \\ &\quad +\left[\frac{\alpha\theta\pi^{\theta(1-\varepsilon)}s}{s}\right](\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\left[\frac{\alpha\pi^{\theta(1-\varepsilon)}s}{s}\right]\hat{s}_{t-1} \end{aligned}$$

and given (75) and (74)

$$\hat{s}_t=\left(1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right)\theta\left(\hat{\phi}_t-\hat{\psi}_t\right)+\alpha\bar{\pi}^{\theta(1-\varepsilon)}[\theta(\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\hat{s}_{t-1}] \tag{80}$$

Finally make use of (68) to obtain

$$\begin{aligned}
\hat{s}_t &= \left(1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right)\theta\left(\hat{\phi}_t-\hat{\psi}_t\right)+\alpha\bar{\pi}^{\theta(1-\varepsilon)}[\theta(\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\hat{s}_{t-1}]= \\
&= \left(1-\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right)\theta\left[\frac{-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}\right](\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\alpha\bar{\pi}^{\theta(1-\varepsilon)}[\theta(\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\hat{s}_{t-1}]= \\
&= \theta\left[\frac{-1+\alpha\bar{\pi}^{\theta(1-\varepsilon)}}{\tilde{\Omega}}+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\right](\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1}= \\
&= \frac{\theta}{\tilde{\Omega}}\left[\alpha\bar{\pi}^{\theta(1-\varepsilon)}\left(1+\tilde{\Omega}\right)-1\right](\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1}
\end{aligned}$$

where $\tilde{\Omega}=\left(\frac{\left(1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}\right)}{\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}\right)$. Hence

$$\hat{s}_t=\frac{\theta}{\tilde{\Omega}}\left[\alpha\bar{\pi}^{\theta(1-\varepsilon)}\left(1+\tilde{\Omega}\right)-1\right](\hat{\pi}_t-\varepsilon\hat{\pi}_{t-1})+\alpha\bar{\pi}^{\theta(1-\varepsilon)}\hat{s}_{t-1}. \tag{81}$$

7.5 Generalizing the Taylor principle

7.5.1 To trend inflation

Here we generalize the Taylor principle as discussed in Woodford (2003, chp. 4) to the case of non-zero steady state inflation.

In the case of: (i) ϕ_π,$\phi_Y > 0$; (ii) standard Neo-Keynesian model featuring zero-inflation steady state; (iii) a contemporaneous interest rate rule; the original Taylor principle $\phi_\pi > 1$ has been generalized to

$$\phi_\pi+\frac{(1-\beta)}{\kappa}\phi_Y>1. \tag{82}$$

As stressed by Woodford (2003, chp. 4), the logic is that the **long run multiplier** of $\hat{\pi}$ on $\hat{\imath}$ must exceed one:

$$\frac{\partial\hat{\imath}}{\partial\hat{\pi}}|_{LR}=\phi_\pi+\phi_Y\frac{\partial\hat{Y}}{\partial\hat{\pi}}|_{LR}=\phi_\pi+\frac{(1-\beta)}{\kappa}\phi_Y>1 \tag{83}$$

since given the standard NKPC

$$\hat{\pi}_t=\beta\hat{\pi}_{t+1}+\kappa\hat{Y}_t \tag{84}$$

hence $\frac{\partial\hat{Y}}{\partial\hat{\pi}}|_{LR}=\frac{(1-\beta)}{\kappa}>0$.

Note that in the space $(\phi_\pi;\phi_Y)$ the condition is defined by

$$\phi_Y>\frac{\kappa}{1-\beta}(1-\phi_\pi)$$

which is the line in the Figure 1 correspondent to zero trend inflation: it goes through the point ($\phi_\pi = 1; \phi_Y = 0$) and is (highly) negatively sloped.

Here we show that in our model: (i) $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ depends on trend inflation; (ii) for standard calibration values, $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ turns negative very soon as trend inflation is positive; (iii) for standard calibration values, $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ increases in absolute value as trend inflation increases. As a result, indeed, the left-lateral frontier in Figure 1 coincides with condition (38), generalised to allow for trend inflation.

The model consists of the following equations: (21),(22), (29) and (35). Differentiating (29) we get

$$\begin{aligned} d\hat{s}\left(1-\alpha\bar{\pi}^{\theta}\right) &= \frac{\theta}{\bar{\Omega}}\left[\bar{\pi}-1\right]d\hat{\pi} \\ \frac{d\hat{s}}{d\hat{\pi}} &= \frac{\frac{\theta}{\bar{\Omega}}\left[\bar{\pi}-1\right]}{(1-\alpha\bar{\pi}^{\theta})} = \Theta. \end{aligned} \tag{85}$$

Then (35) yields

$$\sigma_n d\hat{s} + (\sigma_c + \sigma_n)\, d\hat{Y} = d\hat{mc} \tag{86}$$

and putting (85) into (86)

$$\begin{aligned} d\hat{mc} &= \sigma_n \Theta d\hat{\pi} + (\sigma_c + \sigma_n)\, d\hat{Y} \\ \frac{d\hat{mc}}{d\hat{\pi}} &= \sigma_n \Theta + (\sigma_c + \sigma_n)\frac{d\hat{Y}}{d\hat{\pi}}. \end{aligned} \tag{87}$$

Differentiating (22) we obtain

$$\begin{aligned} \frac{d\hat{\psi}}{d\hat{\pi}} &= (1-\sigma_c)\frac{d\hat{Y}}{d\hat{\pi}} + \frac{d\hat{mc}}{d\hat{\pi}} + \frac{\alpha\beta\bar{\pi}^{\theta}\theta}{1-\alpha\beta\bar{\pi}^{\theta}} \\ \frac{d\hat{\psi}}{d\hat{\pi}} &= (1-\sigma_c)\frac{d\hat{Y}}{d\hat{\pi}} + \underbrace{\left[\sigma_n\Theta + (\sigma_c+\sigma_n)\frac{d\hat{Y}}{d\hat{\pi}}\right]}_{\frac{d\hat{mc}}{d\hat{\pi}} \text{ from (87)}} + \frac{\alpha\beta\bar{\pi}^{\theta}\theta}{1-\alpha\beta\bar{\pi}^{\theta}} \\ \frac{d\hat{\psi}}{d\hat{\pi}} &= (1+\sigma_n)\frac{d\hat{Y}}{d\hat{\pi}} + \sigma_n\Theta + \frac{\alpha\beta\bar{\pi}^{\theta}\theta}{1-\alpha\beta\bar{\pi}^{\theta}}. \end{aligned} \tag{88}$$

Then we can substitute the above equation into (21) to get

$$\hat{\pi} = \beta\hat{\pi} + \lambda(\bar{\pi})\hat{mc} + \lambda(\bar{\pi})\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})}\hat{Y} + \lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\hat{\psi}$$

$$d\hat{\pi}\,(1-\beta) = \lambda(\bar{\pi})d\hat{mc} + \lambda(\bar{\pi})\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})}d\hat{Y} + \lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)d\hat{\psi}$$

$$\lambda(\bar{\pi})\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})}\frac{d\hat{Y}}{d\hat{\pi}} = (1-\beta)-\lambda(\bar{\pi})\frac{d\hat{mc}}{d\hat{\pi}}-\lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\frac{d\hat{\psi}}{d\hat{\pi}}$$

$$\lambda(\bar{\pi})\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})}\frac{d\hat{Y}}{d\hat{\pi}} = (1-\beta)-\lambda(\bar{\pi})\left[\sigma_n\Theta+(\sigma_c+\sigma_n)\frac{d\hat{Y}}{d\hat{\pi}}\right]+$$

$$-\lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\left[(1+\sigma_n)\frac{d\hat{Y}}{d\hat{\pi}}+\sigma_n\Theta+\frac{\alpha\beta\bar{\pi}^{\theta}\theta}{1-\alpha\beta\bar{\pi}^{\theta}}\right]$$

$$\frac{d\hat{Y}}{d\hat{\pi}}\left[\lambda(\bar{\pi})\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})}+\lambda(\bar{\pi})(\sigma_c+\sigma_n)+\lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)(1+\sigma_n)\right]$$
$$= (1-\beta)-\lambda(\bar{\pi})\sigma_n\Theta-\lambda(\bar{\pi})\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\left[\sigma_n\Theta+\frac{\alpha\beta\bar{\pi}^{\theta}\theta}{1-\alpha\beta\bar{\pi}^{\theta}}\right].$$

Divide then by $\lambda(\bar{\pi})(\sigma_c+\sigma_n)\equiv\kappa(\bar{\pi})$

$$\frac{d\hat{Y}}{d\hat{\pi}}\left[1+\frac{(1-\bar{\pi})(1-\sigma_c)}{(1-\alpha\beta\bar{\pi}^{\theta})(\sigma_c+\sigma_n)}+\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\frac{1+\sigma_n}{\sigma_c+\sigma_n}\right]$$
$$= \frac{(1-\beta)}{\lambda(\bar{\pi})(\sigma_c+\sigma_n)}-\frac{\sigma_n\Theta}{\sigma_c+\sigma_n}-\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\left[\frac{\sigma_n\Theta}{\sigma_c+\sigma_n}+\frac{\alpha\beta\bar{\pi}^{\theta}\theta}{(1-\alpha\beta\bar{\pi}^{\theta})(\sigma_c+\sigma_n)}\right].$$

Hence in condition (38), $\left.\frac{d\hat{Y}}{d\hat{\pi}}\right|_{LR}$ in the more general model with positive trend inflation and no indexation is given by

$$\frac{d\hat{Y}}{d\hat{\pi}}\left(\frac{\bar{\pi}-\alpha\beta\bar{\pi}^{\theta}}{1-\alpha\beta\bar{\pi}^{\theta}}\right)=\frac{(1-\beta)}{\kappa(\bar{\pi})}-\frac{\sigma_n\Theta}{\sigma_c+\sigma_n}\left[\frac{\bar{\pi}-\alpha\beta\bar{\pi}^{\theta}}{1-\alpha\beta\bar{\pi}^{\theta}}\right]-\left(\frac{\bar{\pi}-1}{1-\alpha\beta\bar{\pi}^{\theta}}\right)\frac{\alpha\beta\bar{\pi}^{\theta}\theta}{(1-\alpha\beta\bar{\pi}^{\theta})(\sigma_c+\sigma_n)}. \tag{89}$$

It is easy to check that putting $\bar{\pi}=1$, one gets the usual $\frac{d\hat{Y}}{d\hat{\pi}}=\frac{(1-\beta)}{\kappa}$ and then condition (37). Putting this expression into (38) and plotting it for different values of trend inflation, we exactly obtain the left-lateral frontier in Figure 1.

8 Tables

Contemporaneous Rule					
	$\bar{\pi} = 0\%$	$\bar{\pi} = 2\%$	$\bar{\pi} = 4\%$	$\bar{\pi} = 6\%$	$\bar{\pi} = 8\%$
			$\phi_i = 0$		
$\varepsilon = 0\%$	9232	4293 (−53.49%)	1363 (−85.23%)	442 (−95.21%)	68 (−99.26%)
$\varepsilon_{PI} = 50\%$	9653	8165 (−15.41%)	6005 (−37.79%)	3551 (−63.21%)	2210 (−77.10%)
$\varepsilon_{LR} = 50\%$	9232	7761 (−15.93%)	5647 (−38.83%)	3160 (−65.77%)	1845 (−80.01%)
$\varepsilon_{PI} = 100\%$	9680	9680	9680	9680	9680

Table 1. The table shows the number of combinations ϕ_π and ϕ_y that deliver a determinate equilibrium and in brackets the percentage reduction relative to the case $\bar{\pi} = 0\%$. It is computed for σ_c=σ_n=1;θ=11,α=0.75 and. β=0.99. Moreover, $\phi_\pi \in [0, 5]$ and $\phi_y \in [-1, 5]$. Step increase: 0.05.

Inertial Contemporaneous Rules					
	$\overline{\pi} = 0\%$	$\overline{\pi} = 2\%$	$\overline{\pi} = 4\%$	$\overline{\pi} = 6\%$	$\overline{\pi} = 8\%$
			$\phi_i = 0.5$		
$\varepsilon = 0\%$	10821	5885 (-45.61%)	2084 (-80.74%)	811 (-92.50%)	225 (-97.92%)
$\varepsilon_{PI} = 50\%$	10894	9446 (−13.29%)	7344 (−32.58%)	4653 (−57.28%)	3005 (−72.41%)
$\varepsilon_{LR} = 50\%$	10821	9249 (−14.52%)	6947 (−35.80%)	4113 (−61.99%)	2530 (−76.61%)
$\varepsilon_{PI} = 100\%$	10890	10890	10890	10890	10890
			$\phi_i = 1$		
$\varepsilon = 0\%$	12197	7818 (−35.90%)	3781 (−69.00%)	2657 (−78.21%)	2239 (−81.64%)
$\varepsilon_{PI} = 50\%$	12109	10838 (−10.49%)	8846 (−26.94%)	6074 (−49.83%)	4307 (−64.43%)
$\varepsilon_{LR} = 50\%$	12197	10995 (−9.85%)	8861 (−27.35%)	5984 (−50.93%)	4293 (−64.80%)
$\varepsilon_{PI} = 100\%$	12100	12100	12100	12100	12100
			$\phi_i = 2$		
$\varepsilon = 0\%$	12220	9607 (−21.38%)	4461 (−63.49%)	2890 (−76.35%)	2299 (−81.18%)
$\varepsilon_{PI} = 50\%$	12220	12174 (−0.37%)	10676 (−12.63%)	7865 (−35.63%)	5405 (−55.76%)
$\varepsilon_{LR} = 50\%$	12220	12191 (−0.23%)	10589 (−13.34%)	7563 (−38.10%)	5190 (−57.52%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220
			$\phi_i = 5$		
$\varepsilon = 0\%$	12220	12190 (−0.24%)	6513 (−46.70%)	3593 (−70.59%)	2498 (−79.55%)
$\varepsilon_{PI} = 50\%$	12220	12220	12220	11459 (−6.22%)	8265 (−32.36%)
$\varepsilon_{LR} = 50\%$	12220	12220	12220	11256 (−7.88%)	7865 (−35.63%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220

Table 2. See note in table 1.

Forward-Looking Rule					
	$\overline{\pi} = 0\%$	$\overline{\pi} = 2\%$	$\overline{\pi} = 4\%$	$\overline{\pi} = 6\%$	$\overline{\pi} = 8\%$
	$\phi_i = 0$				
$\varepsilon = 0\%$	3028	2326 (−23.18%)	1022 (−66.24%)	280 (−90.75%)	20 (−99.33%)
$\varepsilon_{PI} = 50\%$	3668	3368 (−8.17%)	2980 (−18.7%)	2460 (−32.9%)	1719 (−53.13%)
$\varepsilon_{LR} = 50\%$	3028	2846 (−6.01%)	2552 (−15.71%)	2118 (−30.05%)	1470 (−51.45%)
$\varepsilon_{PI} = 100\%$	5552	5552	5552	5552	5552
	$\phi_i = 0.5$				
$\varepsilon = 0\%$	5920	4163 (−29.67%)	1629 (−72.48%)	587 (−90.08%)	143 (−97.58%)
$\varepsilon_{PI} = 50\%$	6631	5973 (−9.92%)	5118 (−22.81%)	3959 (−40.29%)	2538 (−61.72%)
$\varepsilon_{LR} = 50\%$	5920	5340 (−9.79)	4533 (−23.42)	3406 (−42.46)	2070 (−65.03)
$\varepsilon_{PI} = 100\%$	7229	7229	7229	7229	7229
	$\phi_i = 1$				
$\varepsilon = 0\%$	9436	6349 (−32.71%)	2586 (−72.59%)	1200 (−87.28%)	540 (−94.27%)
$\varepsilon_{PI} = 50\%$	9830	8914 (−9.31%)	7550 (−23.19%)	5582 (−43.21%)	3633 (−63.04%)
$\varepsilon_{LR} = 50\%$	9436	8493 (−9.99%)	7007 (−25.74%)	4946 (−47.58%)	3118 (−66.95%)
$\varepsilon_{PI} = 100\%$	10106	10106	10106	10106	10106
	$\phi_i = 2$				
$\varepsilon = 0\%$	12220	9607 (−21.38%)	4461 (−63.49%)	2890 (−76.35%)	2299 (−81.18%)
$\varepsilon_{PI} = 50\%$	12220	12174 (−0.37%)	10676 (−12.63%)	7865 (−35.63%)	5405 (−55.76%)
$\varepsilon_{LR} = 50\%$	12220	12191 (−0.23%)	10589 (−13.34%)	7563 (−38.10%)	5190 (−57.52%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220
	$\phi_i = 5$				
$\varepsilon = 0\%$	12220	12190 (−0.24%)	6513 (−46.70%)	3593 (−70.59%)	2498 (−79.55%)
$\varepsilon_{PI} = 50\%$	12220	12220	12220	11459 (−6.22%)	8265 (−32.36%)
$\varepsilon_{LR} = 50\%$	12220	12220	12220	11256 (−7.88%)	7866 (−35.63%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220

Table 3. See note in table 1.

Lagged Interest Rule					
	$\overline{\pi}=0\%$	$\overline{\pi}=2\%$	$\overline{\pi}=4\%$	$\overline{\pi}=6\%$	$\overline{\pi}=8\%$
			$\phi_i=0$		
$\varepsilon=0\%$	5429	8563 (57.72%)	8204 (51.11%)	6841 (26.00%)	6265 (15.39%)
$\varepsilon_{PI}=50\%$	5771	6708 (16.23%)	8039 (39.29%)	9290 (60.97%)	8915 (54.47%)
$\varepsilon_{LR}=50\%$	5429	6417 (18.19%)	7851 (44.61%)	9080 (67.25%)	8580 (58.04%)
$\varepsilon_{PI}=100\%$	5958	5958	5958	5958	5958
			$\phi_i=0.5$		
$\varepsilon=0\%$	7392	8911 (20.54%)	7437 (0.60%)	6375 (−13.75%)	5640 (−23.70%)
$\varepsilon_{PI}=50\%$	7494	7917 (5.64%)	8563 (14.26%)	9088 (21.27%)	7947 (6.04%)
$\varepsilon_{LR}=50\%$	7392	7858 (6.30%)	8566 (15.88%)	9053 (22.47%)	7861 (6.34%)
$\varepsilon_{PI}=100\%$	7563	7563	7563	7563	7563
			$\phi_i=1$		
$\varepsilon=0\%$	10040	8849 (−11.86%)	5885 (−41.38%)	4744 (−52.74%)	4319 (−56.98%)
$\varepsilon_{PI}=50\%$	10072	9686 (−3.83%)	9165 (−9.00%)	8292 (−17.67%)	6551 (−34.95%)
$\varepsilon_{LR}=50\%$	10040	9658 (−3.80%)	9094 (−9.42%)	8109 (−19.23%)	6403 (−36.22%)
$\varepsilon_{PI}=100\%$	9924	9924	9924	9924	9924
			$\phi_i=2$		
$\varepsilon=0\%$	12220	9607 (−21.38%)	4461 (−63.49%)	2890 (−76.35%)	2299 (−81.18%)
$\varepsilon_{PI}=50\%$	12220	12174 (−0.37%)	10676 (−12.63%)	7865 (−35.63%)	5405 (−55.76%)
$\varepsilon_{LR}=50\%$	12220	12191 (−0.23%)	10589 (−13.34%)	7563 (−38.10%)	5190 (−57.52%)
$\varepsilon_{PI}=100\%$	12220	12220	12220	12220	12220
			$\phi_i=5$		
$\varepsilon=0\%$	12220	12190 (−0.24%)	6513 (−46.70%)	3593 (−70.59%)	2498 (−79.55%)
$\varepsilon_{PI}=50\%$	12220	12220	12220	11459 (−6.22%)	8265 (−32.36%)
$\varepsilon_{LR}=50\%$	12220	12220	12220	11256 (−7.88)	7865 (−35.63)
$\varepsilon_{PI}=100\%$	12220	12220	12220	12220	12220

Table 4. See note in table 1.

Hybrid Interest Rules (type 1)					
	$\overline{\pi} = 0\%$	$\overline{\pi} = 2\%$	$\overline{\pi} = 4\%$	$\overline{\pi} = 6\%$	$\overline{\pi} = 8\%$
			$\phi_i = 0$		
$\varepsilon = 0\%$	8425	3709 (-55.97%)	1013 (-87.97%)	268 (-96.81%)	20 (-99.76%)
$\varepsilon_{PI} = 50\%$	8755	7310 (-16.50%)	5239 (-40.15%)	2924 (-66.60%)	1717 (-80.38%)
$\varepsilon_{LR} = 50\%$	8425	6938 (-17.64%)	4755 (-43.56%)	2461 (-70.78%)	1364 (-83.81%)
$\varepsilon_{PI} = 100\%$	9780	9780	9780	9780	9780
			$\phi_i = 0.5$		
$\varepsilon = 0\%$	10263	5313 (-48.23%)	1711 (-83.32%)	615 (-94.00%)	149 (-98.54%)
$\varepsilon_{PI} = 50\%$	10596	9105 (-14.07%)	6947 (-34.43%)	4244 (-59.94%)	2652 (-74.97%)
$\varepsilon_{LR} = 50\%$	10263	8653 (-15.68%)	6379 (-37.84%)	3641 (-64.52%)	2156 (-78.99%)
$\varepsilon_{PI} = 100\%$	10846	10846	10846	10846	10846
			$\phi_i = 1$		
$\varepsilon = 0\%$	12197	7818 (-35.90%)	3781 (-69.00%)	2657 (-78.21%)	2239 (-81.64%)
$\varepsilon_{PI} = 50\%$	12128	10861 (-10.44%)	8872 (-26.84%)	6111 (-49.61%)	4351 (-64.12%)
$\varepsilon_{LR} = 50\%$	12197	10995 (-9.85%)	8861 (-27.35%)	5984 (-50.93%)	4293 (-64.80%)
$\varepsilon_{PI} = 100\%$	12100	12100	12100	12100	12100
			$\phi_i = 2$		
$\varepsilon = 0\%$	12220	9607 (-21.38%)	4461 (-63.49%)	2890 (-76.35%)	2299 (-81.18%)
$\varepsilon_{PI} = 50\%$	12220	12174 (-0.37%)	10676 (-12.63%)	7865 (-35.63%)	5405 (-55.76%)
$\varepsilon_{LR} = 50\%$	12220	12191 (-0.23%)	10589 (-13.34%)	7563 (-38.10%)	5190 (-57.52%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220
			$\phi_i = 5$		
$\varepsilon = 0\%$	12220	12190 (-0.24%)	6513 (-46.70%)	3593 (-70.59%)	2498 (-79.55%)
$\varepsilon_{PI} = 50\%$	12220	12220	12220	11459 (-6.22%)	8265 (-32.36%)
$\varepsilon_{LR} = 50\%$	12220	12220	12220	11256 (-7.88%)	7865 (-35.63%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220

Table 5. See note in table 1.

Hybrid Interest Rules (type 2)					
	$\overline{\pi} = 0\%$	$\overline{\pi} = 2\%$	$\overline{\pi} = 4\%$	$\overline{\pi} = 6\%$	$\overline{\pi} = 8\%$
			$\phi_i = 0$		
$\varepsilon = 0\%$	4690	3299 (−29.65%)	1347 (−71.27%)	438 (−90.66%)	71 (−98.48%)
$\varepsilon_{PI} = 50\%$	5083	4657 (−8.38%)	4078 (−19.77%)	3285 (−35.37%)	2214 (−56.44%)
$\varepsilon_{LR} = 50\%$	4690	4190 (−10.66%)	3598 (−23.28%)	2844 (−39.36%)	1838 (−60.81%)
$\varepsilon_{PI} = 100\%$	5148	5148	5148	5148	5148
			$\phi_i = 0.5$		
$\varepsilon = 0\%$	7481	5005 (−33.09%)	1936 (−74.12%)	749 (−89.98%)	212 (−97.16%)
$\varepsilon_{PI} = 50\%$	7618	6926 (−9.08%)	5966 (−21.68%)	4584 (−39.82%)	2984 (−60.82%)
$\varepsilon_{LR} = 50\%$	7481	6616 (−11.56%)	5470 (−26.88%)	3904 (−47.81%)	2389 (−68.06%)
$\varepsilon_{PI} = 100\%$	7660	7660	7660	7660	7660
			$\phi_i = 1$		
$\varepsilon = 0\%$	10572	6975 (−34.02%)	2805 (−73.46%)	1314 (−87.57%)	583 (−94.48%)
$\varepsilon_{PI} = 50\%$	10512	9514 (−9.49%)	8004 (−23.85%)	5800 (−44.82%)	3887 (−63.02%)
$\varepsilon_{LR} = 50\%$	10572	9499 (−10.14%)	7757 (−26.62%)	5236 (−50.47%)	3342 (−68.38%)
$\varepsilon_{PI} = 100\%$	10540	10540	10540	10540	10540
			$\phi_i = 2$		
$\varepsilon = 0\%$	12220	9607 (−21.38%)	4461 (−63.49%)	2890 (−76.35%)	2299 (−81.18%)
$\varepsilon_{PI} = 50\%$	12220	12174 (−0.37%)	10676 (−12.63%)	7865 (−35.63%)	5405 (−55.76%)
$\varepsilon_{LR} = 50\%$	12220	12191 (−0.23%)	10589 (−13.34%)	7563 (−38.10%)	5190 (−57.52%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220
			$\phi_i = 5$		
$\varepsilon = 0\%$	12220	12190 (−0.24%)	6513 (−46.70%)	3593 (−70.59%)	2498 (−79.55%)
$\varepsilon_{PI} = 50\%$	12220	12220	12220	11459 (−6.22%)	8265 (−32.36%)
$\varepsilon_{LR} = 50\%$	12220	12220	12220	11256 (−7.88%)	7866 (−35.63%)
$\varepsilon_{PI} = 100\%$	12220	12220	12220	12220	12220

Table 6. See note in table 1.

9 Figures

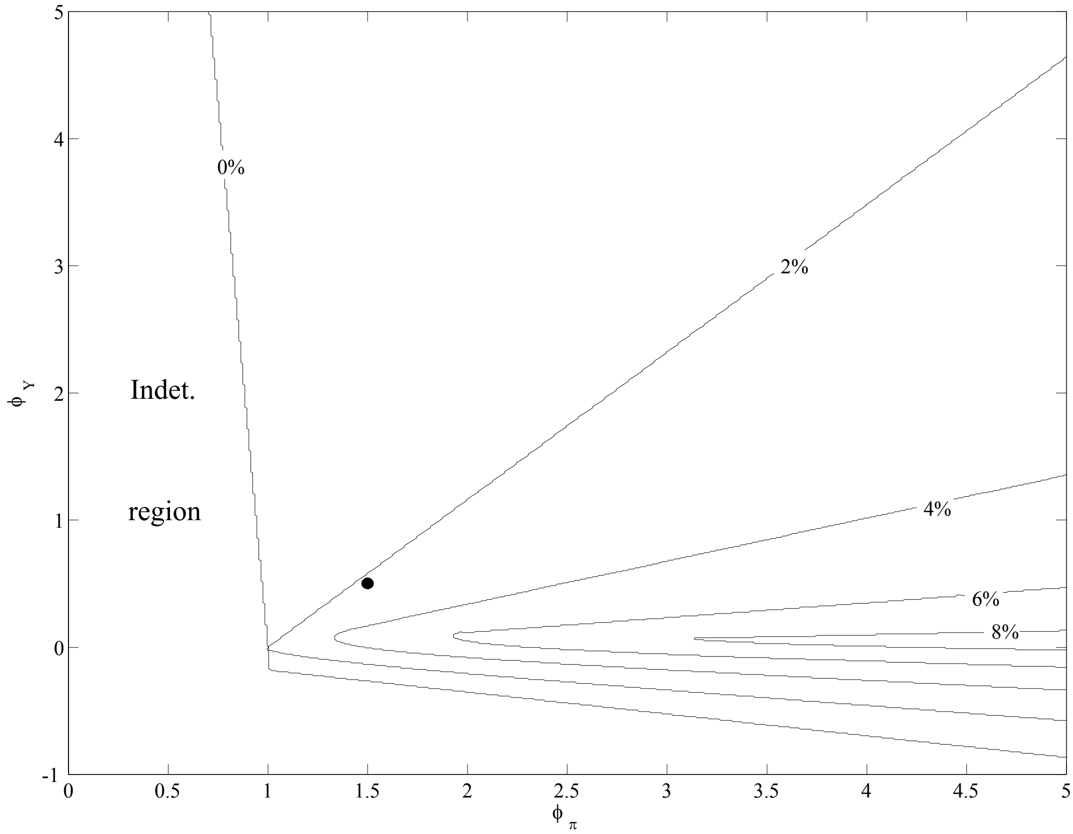

Figure 1: Contemporanous interest rate rule and the effects of trend inflation. The black dot marks the canonical Taylor rule, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

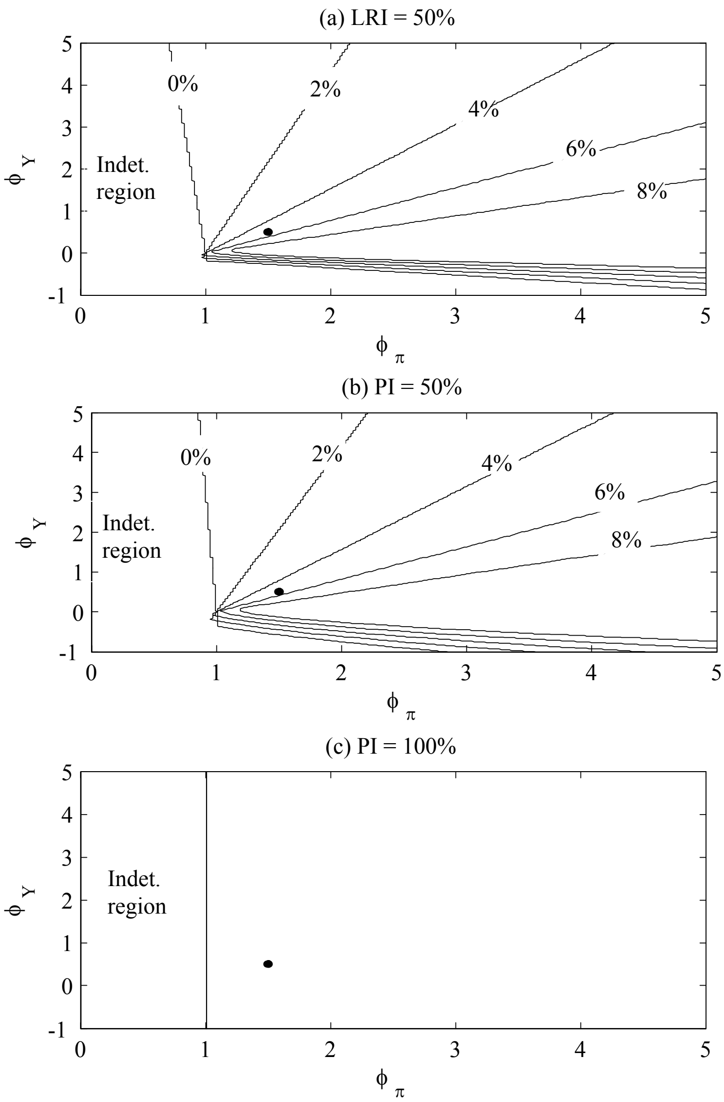

Figure 2: Contemporaneous rule and indexation

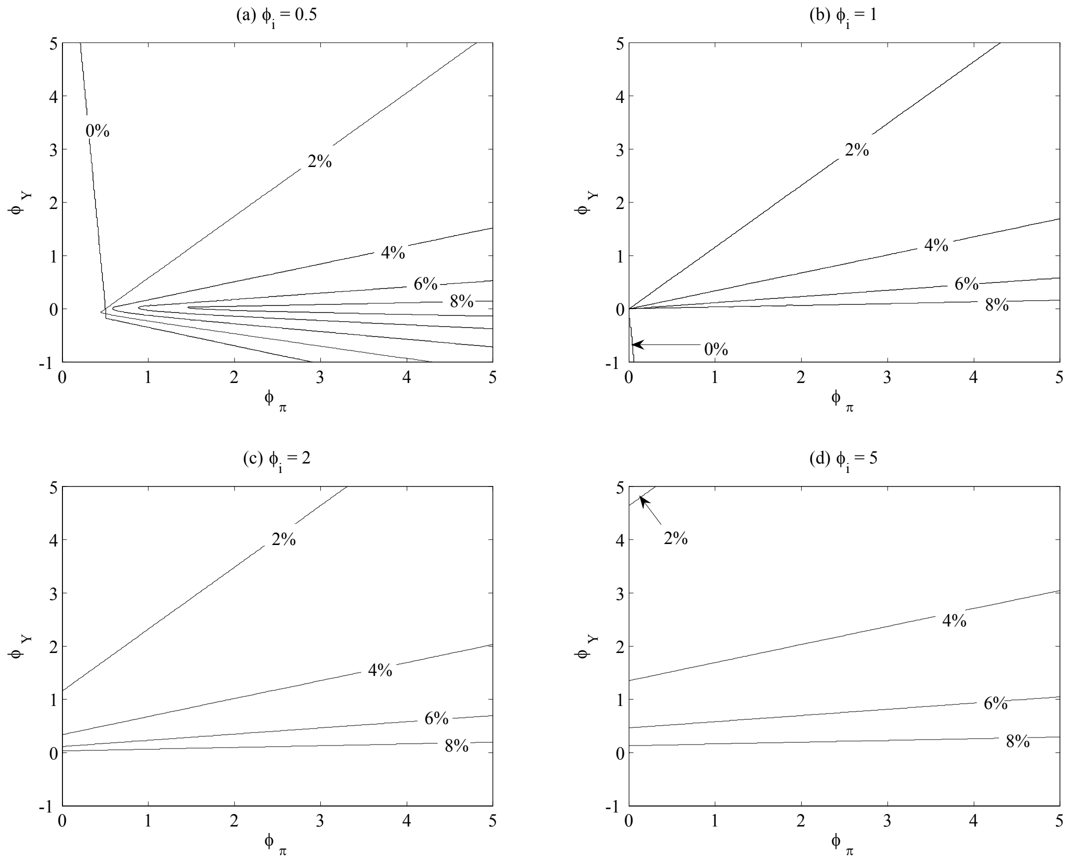

Figure 3: Inertial contemporaneous rule

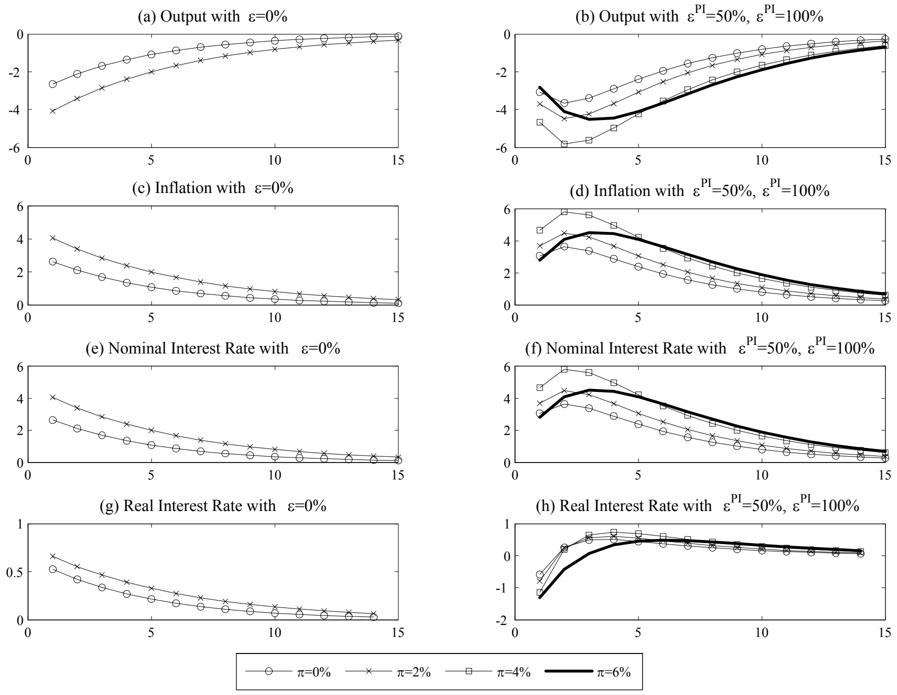

Figure 4: Impulse response function to unit cost-push shock ($\phi_\pi = 1.5$ and $\phi_Y = 0.5$).

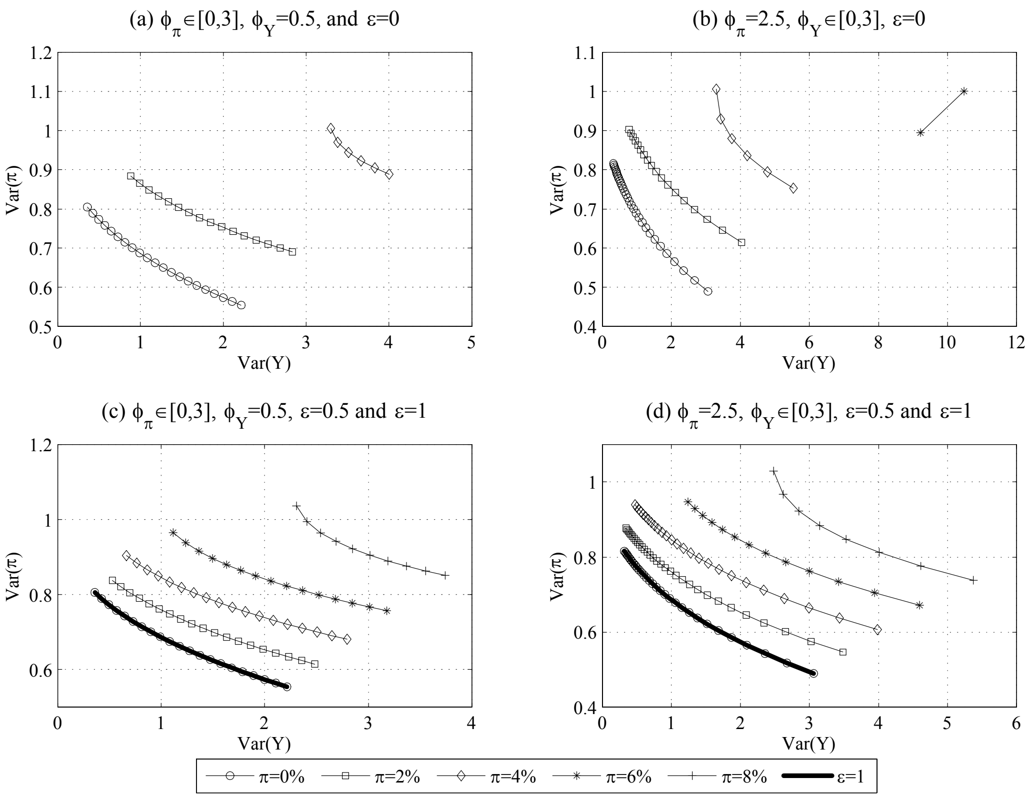

Figure 5: Efficiency frontiers for the contemporaneous interest rate rule with long run inflation indexation.

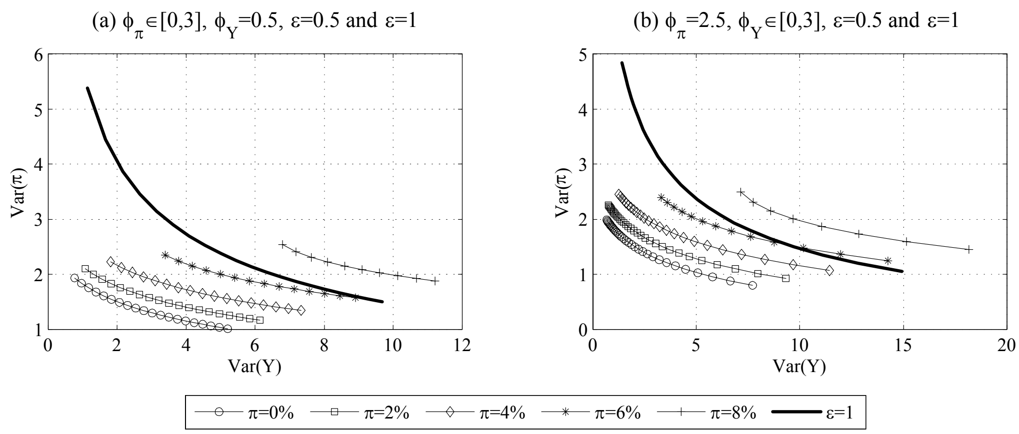

Figure 6: Efficiency frontiers for the contemporaneous interest rate rule with past inflation indexation.

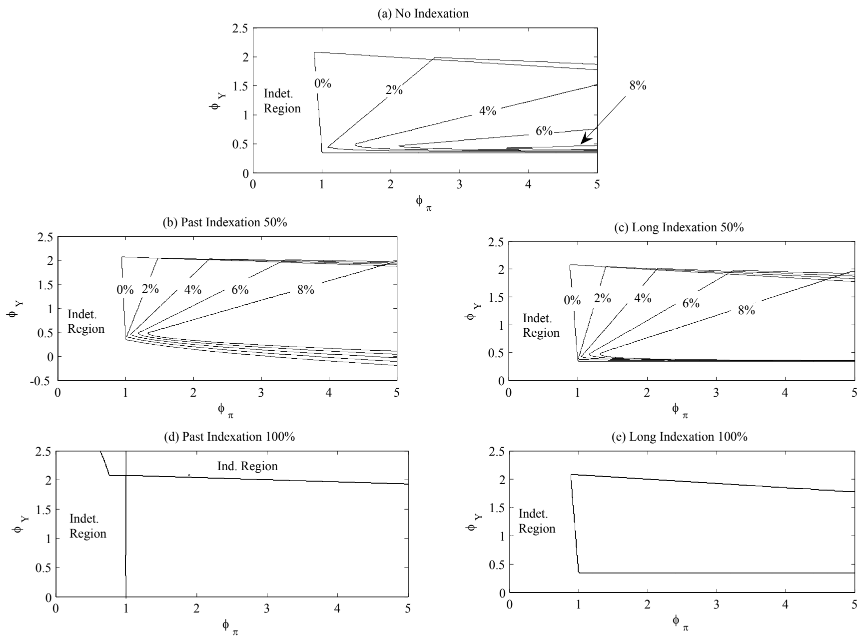

Figure 7: Forward looking rule

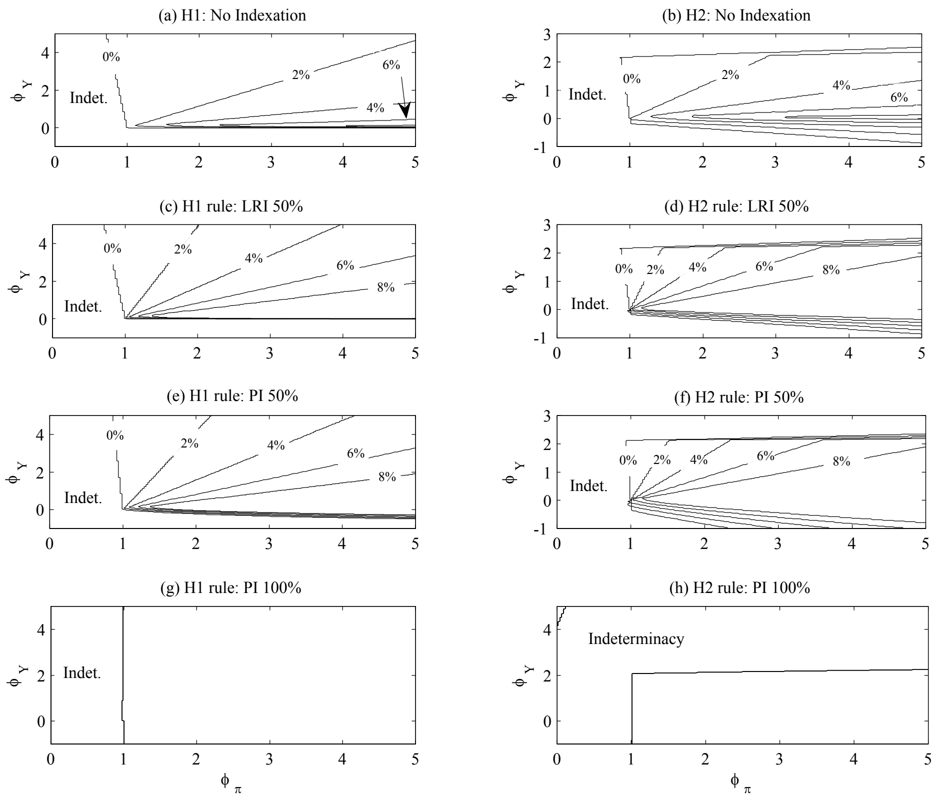

Figure 8: Hybrid interest rate rules

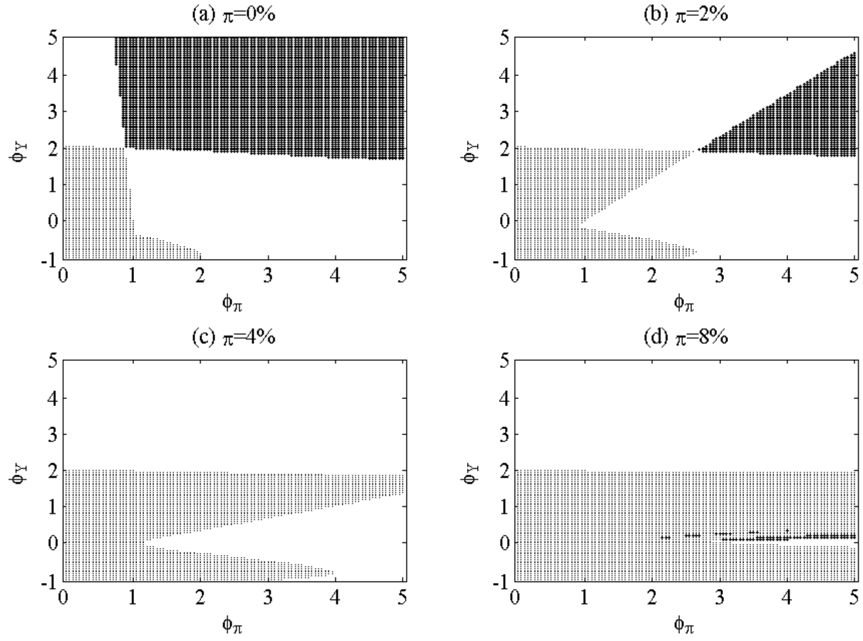

Figure 9: Lagged interest rate rule. Black = instability; grey = indeterminacy; white = determinacy.

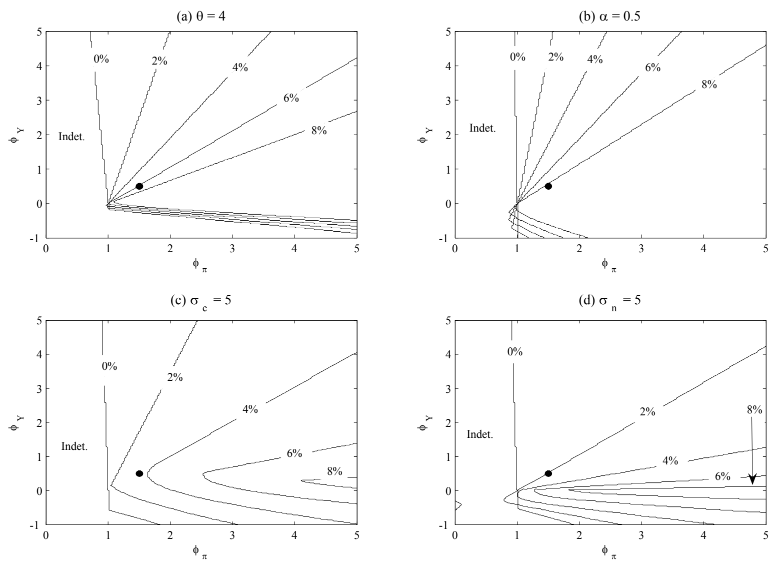

Figure 10: Sensitivity analysis (I)

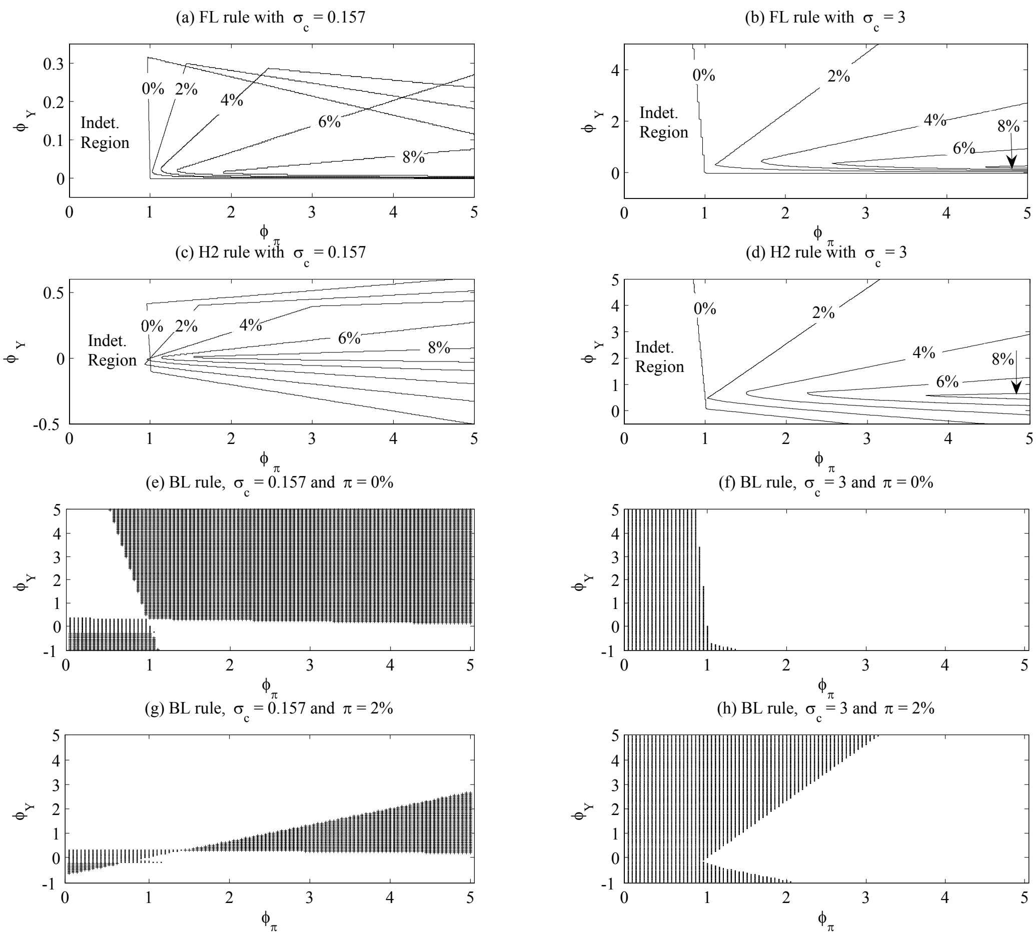

Figure 11: Sensitivity analysis (II)