# THE EFFECT OF SOCIAL INTERACTION AND HERD BEHAVIOUR ON THE FORMATION OF AGENT EXPECTATIONS

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ABSTRACT

*Agent expectations appear to experience inertia, remaining relatively stable for protracted periods punctuated with the occasional structural shift initiated by exogenous changes. The data is also characterised with an underlying level of volatility which varies over time. This paper examines if social interaction and herd behaviour, based on the social learning literature, can provide some insight into the underlying forces behind this dynamic process. The social learning takes place in a network with small world characteristics.*

*Moving from an ordered to a small world network dramatically increases the level of volatility and it quickly reaches a higher level (at which point increasing the randomness of the network has little effect). Assuming that all social networks have small world characteristics then there is an inherent level of volatility in expectations formation. Increasing the influence of experts, by increasing the number of connections from these agents, also increases volatility. This may explain the variability in volatility over time. Finally, it is found that under certain network structures, where the number of connections between agents is increased, herd behaviour leads to information cascades potentially leading to the formation of speculative bubbles.*

Keywords: Social Learning, Expectations Formation, Social Networks, Herd Behaviour, Information Contagion, Volatility, Structural Shifts.

## 1 INTRODUCTION

Agent expectations appear to experience inertia, remaining stable for protracted periods but then undergoing structural shifts at which point expectations continue on a stable trajectory at the new level (Flieth & Foster 2002). During these periods of stability expectations continue to be volatile and noisy but around a stable mean. We examine surveys of the expectations of future business/economic conditions and find that, for the majority of structural shifts, there are significant events, exogenous to the business cycle, occurring during the same time period. (it is found that this result holds whether the agents are consumers or business figures). This paper examines if social interaction and herd behaviour, modelled within a multi-agent framework, can explain this dynamic process of expectation formation.

Within the multi-agent framework economic outcomes can emerge from motives driven by agents' expectations. In turn agents' expectations are formed from simple decision making rules within the self organisational framework (Foster 2000), (Miller & Page 2004), (Tesfatsion 2005). In this paper the stock market is often used to provide an economic context however the model is very general and can be applied to numerous settings. The core of the model is based on the social learning framework initially developed by BHW (1992), hereafter BHW, and Banerjee (1992). The social learning takes place in a ring lattice network consistent with the work on small worlds by Watts (1999) and more recently in an economic context by Cowan and Jonard (2004) which was then extended by Morone & Taylor (2004).

Social interaction and learning is an important phenomenon. For example, within a financial market setting Hong, Kubic and Stein (2004) find that social households[1] are more likely to invest in the stock market. This implies that social interaction is important whether it is simply a talking point (enjoyment derived by talking about the stock market with peers) or a means of social learning (such as sharing knowledge on the benefits of investing in general or sharing information on individual stocks). Also, much of what stock market traders do is based on the general perception of the market not on their own personal beliefs, whether their beliefs match the general perception or not (Froot, Scharftstein & Stein 1992). Ivkovich & Weisbenner (2004) find that a ten percent increase in neighbours purchases is associated with a two percent increase in the households purchase of that stock. Other examples outside of financial markets include the job market (Calvo-Armengol & Zenou 2005), marketing (Chevalier & Mayzlin 2003; Godes & Mayzlin 2004; Money 2000) and law enforcement (Ahn & Suominen 2001).

The basic model consists entirely of herd agents. These agents receive a signal and compare it with the expectations of other agents with whom they are connected, the majority position becomes the expectation of that agent. The signal could represent a private belief formed in a manner consistent with rational expectations. In the absence of heterogenous decision making rules agents enter into an information cascade, learning stops and agents become fixed upon a given set of expectations. While this situation may persist in the short term in the long term such a position is untenable and expectations must inevitably shift. Such a shift can be dramatic indicating a structural shift in the environment (Flieth & Foster 2002). Heterogeneous decision making is introduced with the adding of expert agents. These agents are similar to the fashion leaders and experts discussed in BHW (1992). The addition of one expert agent will enable the population to break the cascade. Exogenous changes in the state of the world now result in structural

[1] Defined as those indicating in the Health and Retirement Study that they interact with their neighbour or attend church.

shifts in agent expectations. The indication that a change in the state of the world has occurred is filtered to herd agents from the expert agents through information contagion.

The effect of introducing the small world properties to the network is then considered. In the absence of the small world (i.e. an ordered network) volatility appears to increase linearly with the number of expert agents. However, in moving away from the ordered network the volatility increases quickly reaching a higher level. At this point increasing the randomness of the network has no effect while increasing the number of experts has minimal effect. Simulations of the small world model reveal patterns in agent expectations with some similar characteristics to those observed in the surveys, specifically protracted periods of inertia followed by structural shifts initiated by exogenous shocks (in the model this is represented by a change in the state of the world). The simulation of agent expectations is also characterised by levels of volatility consistent with the data.

Next the influence of expert agents is more closely examined. Two possible angles are considered, first the strength of the connections between the expert and herd agents are increased, and second the number of connections from each expert to herd agents is increased. It is found that increasing the strength of connections has little effect on the level of volatility. However, increasing the number of connections has a significant effect independent of the small world properties. This may provide some insight behind the change in the level of volatility in agent expectations over time.

Finally we consider whether the structure of the social network can lead to instances when information cascades form in the presence of heterogeneous decision makers. We find that for $k = 4$ (where agents are connected to two other agents on each side, four in total) agents can enter into an information cascade if the social network is ordered. However, if $k$ is set to six or more agents always enter into an information cascade irrespective of whether the network is ordered or has small world properties. This may explain the situation in financial markets where agents continue to hold a view on the market (for example a view that the market remains in a bull run) despite evidence to the contrary. It can also provide some insight for their sudden collapse in instances where network connections decrease.

The paper is organised as follows. The remainder of this section discusses the related literature. Section two presents graphically some data on the evolution of agent expectations over time. Section three outlines the model including the signal generation process, the framework of the social network and the decision making rules. The fourth section explores the long run properties of the model while the fifth section examines the dynamic properties. The sixth section summarises the paper, draws some conclusions and suggests areas of further work.

### 1.1 Relevant Literature

There are a number of different approaches to modelling the dynamic process of expectation formation. For example Lux (1998) and Brock, Hommes & Wagener (2005) use non-linear dynamics to determine supply and demand and then close the model through an exogenous market maker. In Lux (1998) there are two types of traders fundamentalists and chartists. Chartists can be either optimistic or pessimistic. Pessimistic agents turn optimistic with a probability which is proportional to the percentage of optimists in the market and vis versa for optimists. Brock, Hommes & Wagener (2005) consider a market with heterogeneous buyers and sellers where the weights of each sub-group evolves over time being updated by an evolutionary fitness measure, such as past profits.

A second approach to modelling dynamic evolution of expectations is through a Markov process. In Kijima & Uchida (2005) agents have two attributes (owners and non-owners of assets) and two levels (pessimism and optimism). Trades occur when a pessimistic owner meets an optimistic non-owner (the chance of meeting is represented as a Poisson Process). The transition from pessimistic to optimistic is given by $i \cdot \lambda_d$ where $i$ is an asset owner and $\lambda_d$ is the switching intensity from pessimism to optimism and set exogenously (the opposing transition intensity given by $j \cdot \lambda_u$). Gaio, Kaniovski & Zaninotto (2002) is good example of a Markov process used to model changing opinion (rather than expectations). Here agents initially choose between two products. In each successive round one agent is chosen at random and repeatedly chooses other agents at random (in a similar manner to an urn type of experiment), records their choice of product and then replaces the agent and chooses again. Based on the choices revealed through this process, modelled as a Bernoulli trial, the agent decides whether to stay with their product choice or switch. The probabilities, as determined by solving the Bernoulli process, form the transition matrix.

A third approach introduces the concept of the social network whereby agents only communicate with, and see the actions and sometimes payoffs of, those agents which they have a connection with. Therefore, in formulating their decision, agents use the experience of this subset of society, and possibly their own experiences, in updating their posterior using Bayes Law. Bala & Goyal (1998, pp. 602-3) updates agents posterior belief of the state of the world given knowledge of the action of connected agents at time $t$ and the outcome, or payoff, associated with that action. The payoff depends on the action of the connected individuals and the state of the world. Gale & Kariv (2003) modify Bala & Goyal (1998) by assuming that agents are unable to examine the payoff of the agent and instead use the observable actions to infer the unobservable signal (thus bringing it more into line with the traditional literature on social learning). Agents make their decision simultaneously. Using Bayes Law, agents receive a weight attached to each signal (which in many respects is similar to the level of conviction that an agent has in their information) and compares it to the action taken in the previous round by those agents they have a connection to, and then update their posterior. The results are driven by the value and the relative weight signal as well as the structure of the connection. The results of these papers are discussed and compared to the results of this paper in subsequent sections.

Conceptually this paper uses a similar approach to Bala & Goyal (1998) and Gale & Kariv (2003) in analysing the impact of network architecture on both the long run equilibrium and dynamic properties of agent expectations. The point of differentiation is this model introduces the concepts of small worlds, which is then extended to examine the implications and influence of expert agents by varying the number and strength of connections from expert to other agents. Also, this paper is interested only in the agents expectations, not in the payoffs associated with turning that expectation into an action. So agents do not have perfect memory of all actions undertaken by those agents with which they have a connection, they are only interested in the current expectations of connected agents. Finally, agents' signals are updated every period.

The paper is also related to the literature on Word-of-Mouth particularly Banerjee & Fudenberg (2004) and Ellison & Fudenberg (1995)[2]. In these papers agents sample previous decision makers and, based on this information, decide which choice to follow. Banerjee & Fudenberg (2004) find that, when the samples are representative draws from the prevailing distribution in society and each agent samples two or more previous decision makers, then the information cascade develops. If the additional information is informative enough to outweigh the prior then agents herd on the correct decision.

Ellison & Fudenberg (1995) show that if the payoffs from the products are the same then the outcome is dependent on the number of agents *n* sampled. If *n* is small players' exhibit conformity and herd around either product, however, when *n* is large then on aggregate agents swing between products. The case where one of the actions has a higher payoff provides an additional force favouring the superior choice thereby increasing the probability of convergence to that choice of action and reducing or eliminating the possibility of converging to the inferior action. As with the case of equal payoffs increasing *n* leads to a diversity of choices among agents. Long run behaviour is determined by how these forces combine. If *n* is large then diversity forces are overwhelming, when *n* is small you get efficient learning as the weak tendency to diversity and strong tendency to the superior choice prevents herding on the inferior choice. The model in our paper departs from both Banerjee & Fudenberg (2004) and Ellison & Fudenberg (1995) in that these papers employ a random sampling technique to determine lines of communication (rather than the small world network used in this paper) and agents view both the decisions and the pay-offs of connected agents rather than just the decisions.

## 2 EVOLUTION OF EXPECTATED BUSINESS CONDITIONS

Surveys of expected business conditions have been conducted by many different organisations for many different countries over time. In some the focus is on consumer expectations while in others it is the business community. In either case business conditions can have a strong impact on investment decisions and confidence. Two surveys are considered, the Michigan Survey on Expected Change in Business Conditions in a Year, and the German IFO Poll expectational data. The former surveys consumers, the latter industry decision makers. Both these surveys are aimed at the level of decision

[2] In support of the Word-of-Mouth literature Ivkovich & Weisbenner (2004) find that one-third to one-half off information diffusion to be from word-of-mouth.

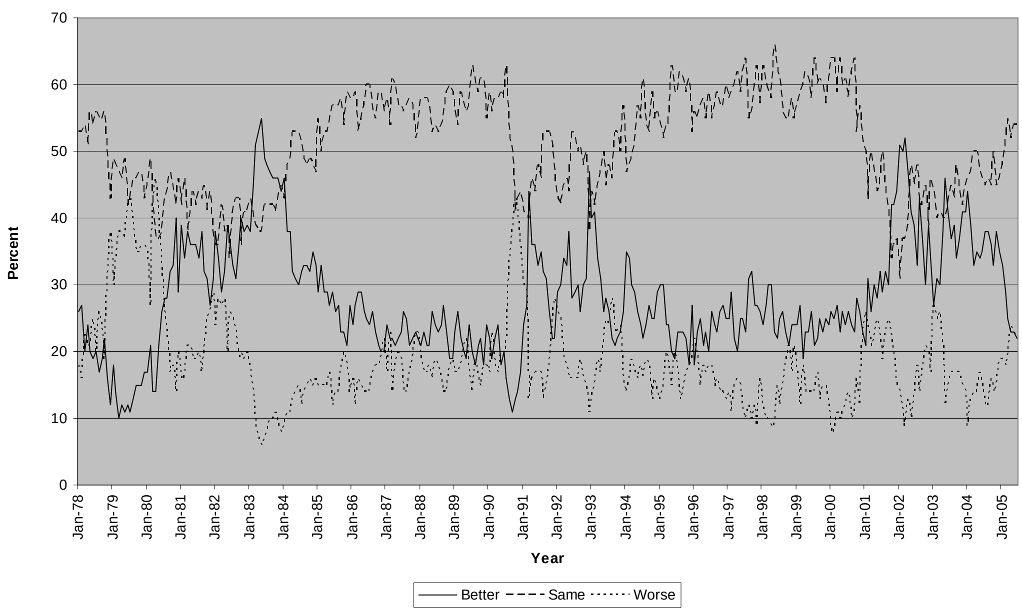
maker rather than experts (or professionals). As a result these surveys are consistent with the theoretical underpinnings of the model proposed in this paper.[3]

The main aim of this section is to illustrate that significant events occurred around the same time that expectations experienced structural shifts. We postulate that these events may have provided some link or impetus to bring about the structural shifts.

## 2.1 Michigan Survey on Expected Change in Business Conditions

The survey data is collected by the Survey Research Center at the University of Michigan and can be found in the University of Michigan website. The survey has been conducted since 1954 first on a half yearly basis, increasing in frequency over time until the survey was conducted monthly from 1978. This specific survey ask US consumers the following question "And how about a year from now, do you expect that in the country as a whole business conditions will be better, or worse than they are at present, or just about the same?" The results are presented in the following figure.

*Figure 1: Michigan Survey on Expected Change in Business Conditions*




Looking at the level of response to each of the answers in turn reveals a striking pattern. Levels of expectation appear to swing between quite well defined highs and lows suggesting the existence of multiple equilibria. For example the percentage of responders that believed the economy would remain about the same oscillates, albeit irregularly, between around 40% and 60%, 'better' between 20% and 40%, and 'worse' between 10% and 30%. Shifts between the two equilibria can be quite sharp and, as will be shown, can be linked to major exogenous events. Between these shifts, or phase transitions,

[3] For example the Livingston Survey on expected inflation would not fall into this category as it is aimed at economists that are more likely to behave in the manner of experts.

expectations can spend a protracted amount of time fluctuating around one of the equilibria. Finally the series exhibits period of extreme volatility such as post January 2001 and also periods of relative calm. The historical information outlined below was sourced from the Wikipedia encyclopaedia[4].

From inspection it appears that expectations undergo phase transitions at a number of dates between 1978 and 2005. The period leading up to the 1980s was dominated by negative sentiment. Withdrawal from the Vietnamese War, and the consequential fall of South Vietnam to communism, the Watergate scandal and the Iranian hostage crises all lead to a diminishing confidence in the American Government. On the economic front America was experiencing a period of stagflation in part as a result of OPEC beginning to exercise its strength in setting oil prices. Carter was elected in 1976 but was unable to reverse the negative sentiment. This was demonstrated in 1979 when Carter gave a nationally televised speech, later to become known as the malaise speech, in which he identified a crisis of confidence in the American people.

Expectations rose sharply in 1980 with the election of Ronald Reagan. They continued to rise sharply as tax cuts and tight money policy addressed the issue of stagflation (at the expense of large budget deficits, high interest rates and high unemployment). 1982 saw the beginning of the worst recession in America since the great depression however, strangely perhaps, expectations did not fall until the end of 1983 by which time the economy had already began to recover. The downward movement in expectations continued until the beginning of 1986 after which expectations remained stable until 1991 where it again rose sharply (in the final period of 1990 expectations did dive before taking off). The fall in expectations in late 1990 coincided with the Iraq invasion of Kuwait on August 2.

In 1991 the Security Council declared war on Iraq with the US troops forming the majority of the forces. This event coincided with the sharp rise in expectations in that period. However, it was in this period that the US also experienced domestic economic problems which ultimately lead to the election of Bill Clinton in 1992. At this point there was a sharp rise in expectations followed by a brief peak in 1993 followed by a volatile period to the end of 1995. Bill Clinton is regarded as one of the more divisive presidents and this could be one of the underlying causes of the high volatility in this period. In 1995 the Republicans gained control of the House of Representatives. This resulted in compromises in the more extreme policies of the Clinton Administration and could plausibly be behind the reduction in volatility in expectations that followed.

In January 2001 George W Bush was elected into office. This again lead to an upswing in expectations in beginning in 2001 but really taking off in late 2001. The rise however, was short lived. By mid 2002 expectations started to fall dramatically coinciding with the stock market crash of 2002. Interestingly the September 11 2001 terrorist attack did not effect expectations nor the stock market in the medium term. Since 2002 expectations have been on a downward trajectory while at the same time experiencing significant volatility. Part of this could be due to the divisive nature of the Bush policies, including

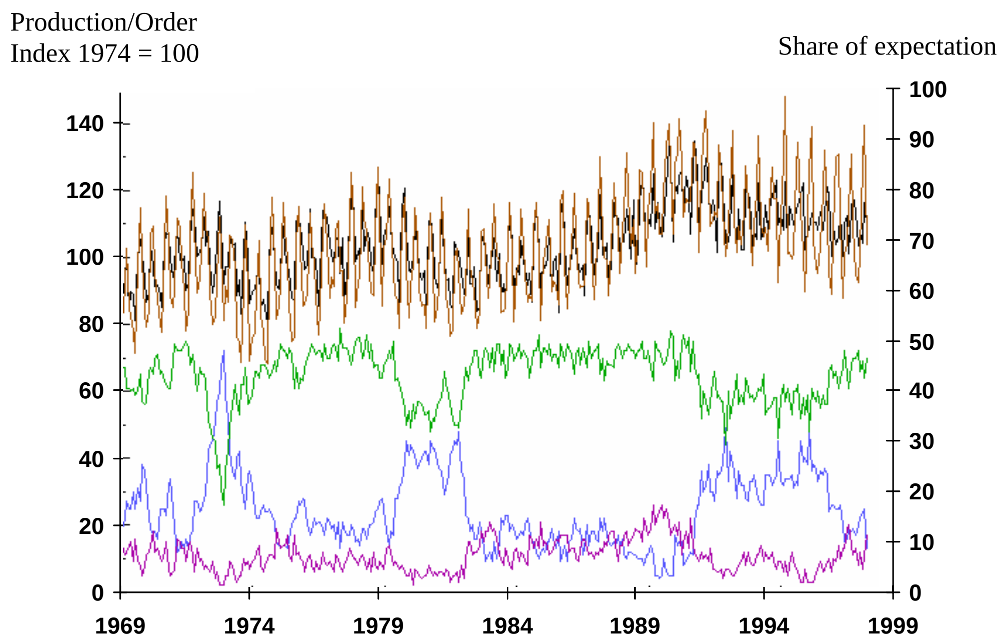
[4] The Wikipedia encyclopaedia can be found at http://en.wikipedia.org/wiki/Main_Page.

the policies on Iraq as well as Federal Government intervention into State Government matters (often these matters being of a religious nature). Also, the world is experiencing rising oil prices and the US has experienced some server natural disasters including the hurricanes Katrina and Rita.

## 2.2 German IFO Poll on Expected Future Business Conditions

The IFO Poll surveys key industrial decision makers asking questions about their expectations on a range of economic variables including prices and business conditions. Flieth & Foster (2002) consider the expected future changes in business climate in the consumer goods minus food and luxuries sector. They find that the series on expectations experience protracted periods at a particular level punctuated by shorter periods of deviation in a manner not dissimilar to that found in the Michigan survey. Figure 1 on page 380 in Flieth & Foster (2002) is reproduced with permission below.

*Figure 2: German IFO Poll on Expected Future Business Conditions*[5]




Flieth & Foster (2002) undertake a careful analysis of the structural shifts in the survey data and find that, like with the Michigan survey, they correspond to significant exogenous events. The 1973 rise in negative sentiment corresponded with the beginning of the oil crises. The period of negative expectations lasted until 1974 at which point neutral expectations dominated until 1980. This point coincided with a period when the ruling government, the Social Democratic government, faced serious political difficulties. Expectations recovered with the election of the Christian Democratic government. The reunification boom in the economy reversed in late 1991 at which point positive and neutral expectations fell and negative expectations rose sharply. Expectations recovered in 1986.

[5] Reproduced from Flieth and Foster (2002) with permission.

## 3 THE MODEL

### 3.1 Overview

The centrepiece of a model of herd behaviour is the coordination mechanism. It comprises of a number of separate components, an observable signal, a social network (defining who is connected to whom) and decision making rules (based on the observable signals what is the agent's decision). Consider the following illustrative. There are $i \in I = \{1, N\}$ agents. At the beginning of each round $t \in [1, T]$ each agent receive a private binary signal $x \in X = (1,0)$ on the state of the world. Each agent *i* would then undertake a process to establish a view on how a market will perform in the next period. They do this by considering the signal they receive as well as the most recent view taken by each of the agent's *j* with which they have a connection. Agent *i's* signal is then adjusted in light of the discussions with connected agents and this becomes their view for round *t*. It is this view that is presented to the market with the private signal never being released. Each component is now considered in more detail.

Note no link between agents' expectations and actual outcomes is made in this paper therefore, there is no need to account for the possibility of agents playing strategic games for gain. However, if agents' expectations were to map to some set of actions then agents could perceivably gain from adopting strategic behaviour. It is unlikely that the herd agent would benefit from such behaviour as there are many agents in this model and each agent comes into contact with only a small subset of neighbours. Expert agents could benefit from such behaviour in the case where connections from these agents are stronger, or there are multiple connections from these agents to herd agents. You could assume that strategic behaviour would not be beneficial in the long run as trust betrothed to these experts would be broken (essentially they would no longer be considered experts), however, the reality is that such behaviour does exist and the model will not be addressing this specific influence.

### 3.2 Generating the Signal

As is common in the social learning literature agents do not know the true state of the world. Instead they form a posterior belief through a Bayesian learning process. Agents receive a private binary signal $x \in X = (1,0)$ which dependent on the state of the world $v \in V = 1,0$. Given X agents attempt to determine V according to:

$$P(V|\boldsymbol{X}) = \frac{P(\boldsymbol{X}|V) \cdot P(V)}{P(\boldsymbol{X})} \tag{1}$$

The value of both the likelihood function and the prior will need to be determined. It is likely that these factors will differ between agents. To complicate matters agents may not follow their own beliefs. For example agents may believe that stocks are overpriced but that the price will continue to rise in the next period or vis versa (Vissing-Jorgensen 2003, p. 10).

In order to focus on the effects of social learning and network structure, rather than the Bayesian learning process a simplified framework is employed whereby agents have identical likelihood and priors according to the following set of rules:

$$P(X = 1|V = 1) = P(X = 0|V = 0) = q > 0.5 \tag{2}$$

$$P(X = 1|v = 0) = P(X = 0|V = 1) = 1 - q \tag{3}$$

$$P(V = 1) = P(V = 0) = 0.5 \tag{4}$$

### 3.3 The Social Network

Consistent with the work on small worlds by Watts (1999) the network will consists of: a population of agents $I$ in some finite social space; and a list of connections between agents (initially defined as either 1 or 0). For any two individuals $i$ and $j$ a connection exists if $\chi(i, j) = 1$ otherwise $\chi(i, j) = 0$. Under this assumption the network is solely concerned with who is connected with whom and is not concerned with the strength of the individual connections. In latter sections the strength between certain agents will be varied to replicate the case where the views of certain agents (such as experts) hold more sway than other agents. This introduces the concept of 'social distance'. Defining social distance can be problematic and difficult to measure as it does not follows the standard Euclidean metric[6]. While defining the network purely as a function of who is connected to whom is on much firmer theoretical and empirical grounds (Watts 1999, p. 22) the influence of experts, through stronger than average ties to many agents, is prevalent in many economic settings including financial markets or protests.

The network is subject to a number of restrictions. Each connection is symmetric and unique with no multiple connections between the same two individuals. Further the number of connections must be significantly smaller than the total number of connections available.

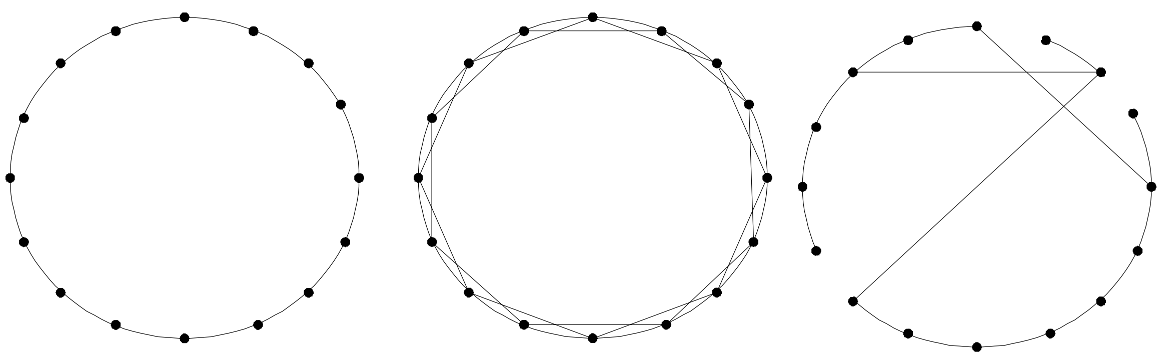
Social systems lie between the completely ordered network and the completely random one. It is therefore a useful starting point to examine the two extremes as reference points and then looking more closely at the results as you move from one to the other. These intermediate solutions provide the small world scenarios. The ring lattice graph provides a useful starting point for the analysis[7]. It has two parameters the number of dimensions $d$ (=1) and nearest neighbours $k$ (> 2).

Each agent $i$ is selected in turn along with the edge to the nearest neighbour in a clockwise sense. The connection is deleted and replaced with a random connection with a pre-determined probability $p$. Each agent goes through this process until all agents have

[6] As an example if agent A knows B and C well, and hence the space between A and B and A and C is small, but B and C do not know each other at all then Euclidean space is violated.
[7] See Watts (1999, p.65) for technical justifications for this choice.

been assessed. The process then repeats itself for the next nearest neighbour if $k$ = 4 and then the next nearest neighbour if $k$ = 6 and so on. In total $k/2$ rounds must be completed.[8] There is no social justification for a model that replaces one connection with another connection at random. However, given the high use of the internet and chat rooms the random approach may not be far from reality.[9] A more rigorous justification would be to compare the characteristics of a network formed using a more realistic assumption with the random process. Watts (1999, p. 69) does this by comparing a network where the likelihood of forming a connection is dependent on the number of common connections an agent has with another agent and finds that the statistical properties of the two approaches are similar and "*are the result of different manifestations of the same mechanism*".

*Figure 4: Ordered Networks and Small Worlds*



$k$ = 2 and $p$ = 0 $k$ = 4 and $p$ = 0 $k$ = 2 and $p$ > 0

### 3.4 Decision Making Rule

In the first round each agent receives a signal according to equation 1 and follows that signal. Therefore, the network does not impact on the expectations of agents in the first round. This is justified as the focus is on the stability of long run equilibria and the dynamics of the steady state. At the end of the first round $t=1$ agents have adopted an expectation $x_i$. Let $\boldsymbol{X_n}$ be the set of opinions of those agents connected to $i$. In the case of a ring lattice with $k = 2$ $\boldsymbol{X_n} = (x_{i-1}, x_i, x_{i+1})$, where: $x_{i-1}$ represents the expectation formed by $i$-$1$ at time $t$, $x_i$ represents the signal received by $i$ at time $t$ and $x_{i+1}$ represents the expectation formed by $i+1$ at time $t$-$1$. The prior probability of $V$ can now be updated by forming the posterior of $V$ given the knowledge gained through conversation according to

[8] The approach adopted in this paper is referred to as the $\beta$ Graph in Watts (1999).

[9] This is certainly the case with financial markets where agents are just as likely to source information from unknown analysts via the web, or communicate to other traders in chat rooms as to talk to friends and neighbours.

$$P_i\left(V\middle|\boldsymbol{X_n}\right)=\frac{P\left(\boldsymbol{X_n}\middle|V\right)\cdot P(V)}{P\left(\boldsymbol{X_n}\right)}$$

Returning to the case of a ring lattice with $k$ = 2, if both agents *i-1* and *i+1* form an expectation that $V$ = 0 and $i$ receives a signal $x_i = 1$ then:

$$P_i\left(V=1\middle|x_{i-1}=0;x_i=1;x_{i+1}=0\right)=\frac{P\left(\boldsymbol{X_n}\middle|V\right)\cdot P(V)}{P\left(\middle|\boldsymbol{X_n}\right)}$$

$$=\frac{P\left(x_i=1\middle|V=1\right)\cdot P\left(V=1\middle|x_{i-1}=0;x_{i+1}=0\right)}{P\left(x_i=1\middle|V=1\right)\cdot P\left(V=1\middle|x_{i-1}=0;x_{i+1}=0\right)+P\left(x_i=1\middle|V=0\right)\cdot P\left(V=0\middle|x_{i-1}=0;x_{i+1}=0\right)}$$

Faced with this scenario and assuming that agents gives equal weight to all $\boldsymbol{X_i}$ then, as $P\left(x_i=1\middle|V=0\right)\cdot P\left(V=0\middle|x_{i-1}=0;x_{i+1}=0\right)>P\left(x_i=1\middle|V=1\right)\cdot P\left(V=1\middle|x_{i-1}=0;x_{i+1}=0\right)$ $i$ will ignore their own signal and update their prior such that the true state of the world is 0. The dynamic properties of the model become:

$$P_{i,t}\left(V_t\middle|\boldsymbol{X}_{n,t}\right)=\frac{P\left(\boldsymbol{X}_{n,t}\middle|V_t\right)\cdot P(V_t)}{P\left(\boldsymbol{X}_{n,t}\right)}\qquad \boldsymbol{X}_{n,t}\subset\boldsymbol{X_t}$$

where $\boldsymbol{X_t}=\{x_{1,t};....;x_{i-1,t};x_{i,t};x_{i+1,t-1};.....;x_{n,t-1}\}$

Agents update their decisions sequentially but make repeated decisions[10]. Further, in updating their prior, herd agents do not take into account their expectations formed in the previous round; only the signal they receive. Essentially the agent starts each time period with a blank sheet of paper and a new signal. This can be justified in instances where the past does not matter (such as fads or fashion) or is captured in the state of the world and consequently in the signal obtained by the agents. For example in the stock market prices incorporate past information with the only concern to agents being the future direction of prices.

This process does not mimic the types of conversations, and social learning, that occurs when individuals meet (for example there will be an element of joint decision making rather than agent $i$ conferring with agent *i+1* prior to formulating a decision, then inturn *i+1* confers with $i$). However, what this approach does do is emphasise the effects of ‘Chinese Whispers’ where, because the communication is by word of mouth, hard evidence is not always provided, nor the full process by which the agent makes their decision (Banerjee & Fudenberg 2004). It also incorporates a form of ‘public weighting’ appropriate to such models.

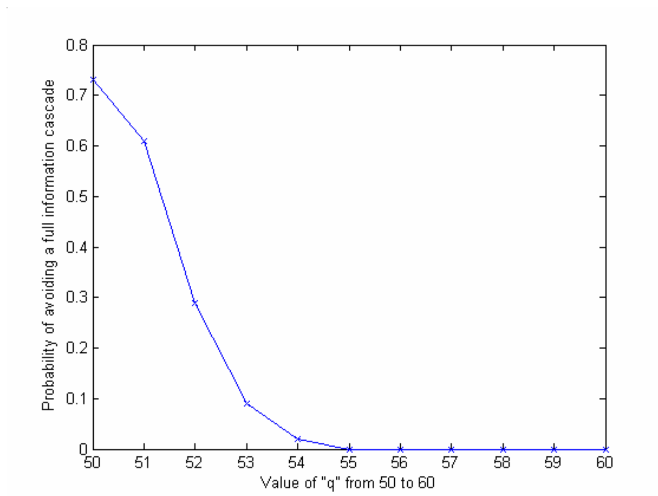
[10] An alternative assumption to assuming an exogenous queue is adopted in Gale & Kariv (2003) where agents simultaneously make their decision with prior knowledge of the actions taken by their neighbours in the previous round.

## 4. LONG RUN EQUILIBRIUM

Consistent with the results of BHW (1992) and Banerjee (1992) when the network consists entirely of herd agents information becomes blocked and all learning ceases. For the purpose of undertaking the numerical analysis the following parameter values are used unless specified otherwise: $N = 200$, $q = 70$ and $k = 2$. In order to test the robustness of these results simulations are also run with N=100 and $q = 60$ and 80 with no noticeable changes to the results. The effect of varying the number of connections between herd agents $k$ is considered in detail in section 5.4.

We examine the probability that a network consisting of 200 agents can avoid an information cascade after 900 rounds.

*Figure 5: Percentage probability that agents avoid an information cascade after 900 rounds (Inverse Hazard Function)*

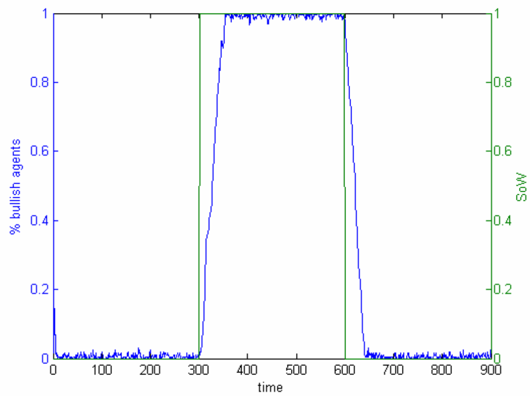



100 trials were run for each increment of $q$ (noting that $q = 50$ represents the case where agents are following a random walk). It confirms that an information cascade forms with a probability of one even for low $q$ (i.e. $q = 50+\varepsilon$). As agents follow their own signal in the first round the probability that agents cascade on the wrong state of the world is negligible. This is consistent with the results of Ellison & Fudenberg (1995) which adopts an exogenous initial state with agents making repeated decisions.

### 4.1 Adding the Expert Agent

Expert agents add another dimension to the decision making process. Experts tend to be high precision individuals that are more inclined to use their own information rather than those that they come into contact with (BHW 1992, p. 1004). For the purpose of numerical analysis the expert agents are spaced evenly within the network (so if there is one expert agent and $N = 200$, the expert agent is set to $i = 100$). Within the framework of BHW (1992) this is equivalent to high precision individuals that make their decision later in the sequence. It is shown that, with the inclusion of one expert, agents always herd around the correct state of the world.

*Figure 6: One Expert Agent*




For $t < 300$ $V$ is set exogenously to 0. In a financial markets context this could represent a bear market, or a price less then fundamental value. As can be seen agents quickly herd around $x = 0$. At $t = 301$ $V$ is changed to 1 representing a structural change in the system. Within a short period of time agents switch their belief of $v$ to 1 (i.e. all but a few agents hold that $x = 1$ at any point in time). At $t = 601$ $V$ is again changed and the same result occurs.

The outcomes of the model has some similarity with that of BHW (1992) in that the presence of an expert, when they appear later in the sequence, has the potential to break cascades but cascades may still be incorrect. In our model experts not only break cascades but the herd always forms on the correct state of the world in finite time. The key difference lies in the basic assumptions of the two models. In BHW (1992) agents are aware that they are in information cascade and that all actions taken by individuals following the commencement of the cascade contain no additional information. Therefore, the only actions that an individual takes into account are those of decision makers prior to the commencement of the cascade and the experts because these are the only actions from which some information about the signal can be gleaned. In our model no such assumptions are necessary. Instead the presence of experts ensures that information always flows to all agents through contagion as they make repeated decisions over time. These results are consistent with Banerjee & Fudenberg (2004) where agents make a one off decision but some agents observe ‘uninformed histories’ requiring them to rely on their own signal. As connections between agents are derived from random draws the end result is that the probability that agents will make a correct decision at a very late date is close to one. The results are also consistent with those of Bala & Goyal (1998, p. 604) who find for that even if one agent eventually learns to chose ex post optimal actions then so will the rest of society. However, it must be remembered that payoffs as well as actions are observable in their model.

## 5 DYNAMIC PROPERTIES

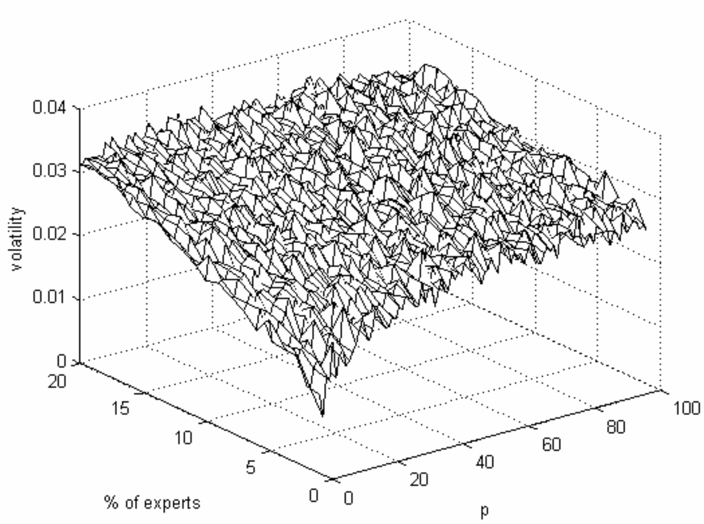
Here our focus is on the level of volatility in agent expectations and the mean number of agents that disagree with the state of the world. It is in this section that we also examine the effect of introducing small world properties to the social network. As can be seen from Figure 6 the speed of convergence of expectations is rapid with agents quickly adjusting their perceptions as information on changes in the state of the world spread through the social network. This is consistent with the results of both (Bala & Goyal 1998) and (Gale & Kariv 2003). Adding more expert agents increases the speed of convergence as well as the level of volatility.

### 5.1 Small World Properties of the Social Network

Firstly we consider the level of volatility as you increase the level of randomness *p* and the number of expert agents. Volatility is measured as the standard deviation with $\sigma = 1$. The results are presented in Figure 7.

As can be seen from the diagrams below for *p* approximately equal to 0 the number of experts affects the level of volatility in what appears to be a linear fashion (steadily increasing from 0.1 with one agent to 0.05 when 10 expert agents are present). As you increase *p* the level of volatility rises steadily in a logistic manner before reaching a plateau for around *p* equal to 40%. At this point increases in either the number of expert agents, or the level of randomness (but holding *k* constant and equal to 2), has very little effect on the level of volatility. Assuming that $p > 10\%$ for all social networks then there is an inherent level of volatility in agent expectations. At this point over half of the level of volatility associated with a random graph has been reached.

*Figure 7: Volatility as you increase the number of expert agents and p*

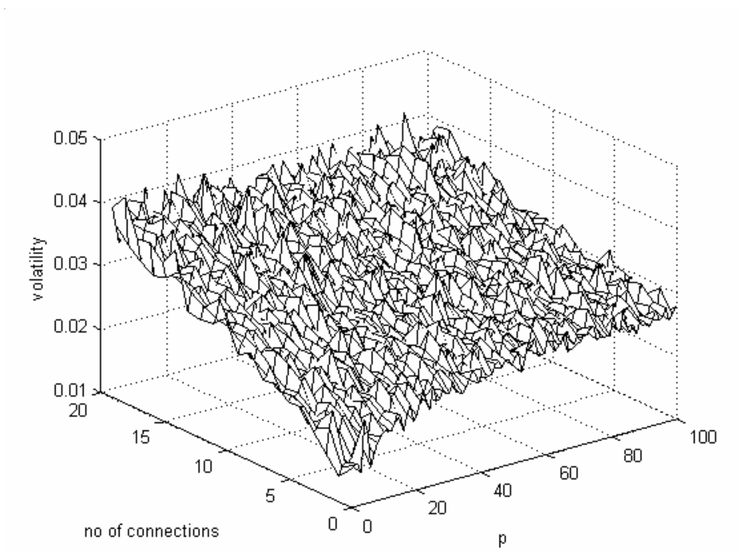


### 5.2 The Power of Expert Agents

As noted earlier expert agents are high precision individuals that tend to use their own information. However, experts also tend to have an increased influence over other agents. Experts are important because they provide valuable information particularly where that information is difficult to obtain or process or drawing conclusions is subjective. Two types of experts are considered in this paper. The first are experts that are well respected in the general community and are connected to many other agents in the network. There are many examples of such experts including financial gurus such as Warren Buffet or Allan Greenspan, or community leaders such as the rock star "Bono"[11] (who has had such an influence in convincing western governments to forgive third world debt). The heads of governments, private companies or other institutions (for example Greenpeace) could also take the mantle of experts. These are represented in the model as agents who have one way connections with many agents. The second type of agent is one whom is recognised locally as an expert. A good example of such an expert might be the local doctor or a financial planner. In the model these agents have the same number of connections as the herd agent but the strength of their connections is twice that of the herd agent.

In what follows three percent of agents are experts and as you increase the number of connections from the experts it is found that volatility increases. Unlike the previous case where the number of experts agents is increased, the effect of increasing the number of connections persists strongly for $p$ > 40% i.e. the volatility associated with this increases is in addition to the volatility due to the small world effect.

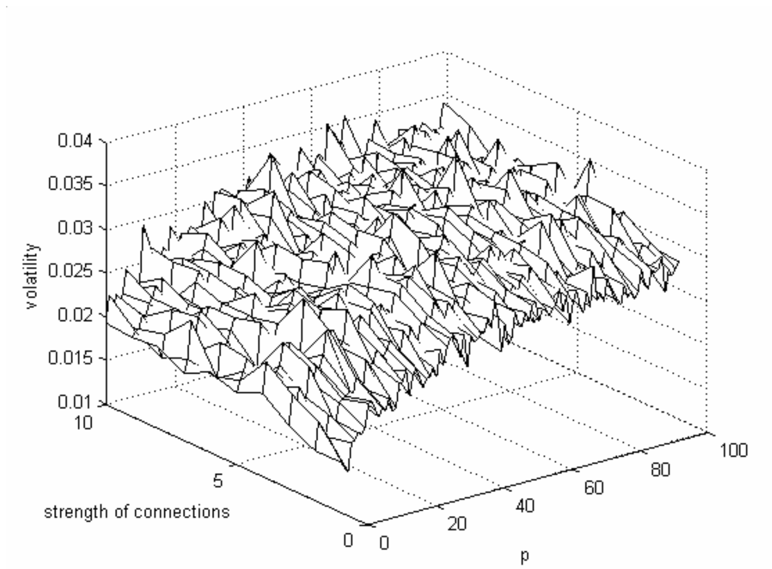
*Figure 8: Volatility as you increase the number of connections from expert agents*




[11] Birth name Paul Hewson.

The results as illustrated by Figures 7 and 8 seem to exhibit some similar characteristics with the survey data presented in Figures one to three where you have an underlying level of volatility across the total timeframe punctuated with periods of high volatility and structural shifts. The periods of high volatility coming during times of high debate and high interest as experts increase their coverage within the social network through the media and other sources. Further structural shifts are the result of exogenous changes in the state of the world. Returning back to Figure 1 the expectations series becomes volatile between 1981 and 1983, 1993 and 1995 and from 2002 onwards. The first period was dominated by Regan and supply side policies. These policies generated much debate and concern from some quarters of economists particularly relating to the effect of reducing taxes on government debt. It is possible that it was this debate, and the uncertainty in economic prosperity that it caused, which drove the high level of volatility. The second period of 1993 to 1995 could well have been driven by the divisive and extreme policies of the Clinton Administration. As in the previous volatile period there were concerns that the policies could result in larger government deficits particularly those focusing on social equity. The final period has been dominated by division including the stance on America's involvement in the War in Iraq. The steady and rather steep rise in the price of oil, and the ensuing debate on whether this rise is just a phase or a more permanent phenomenon, could also be effecting expectations on future economic conditions.

*Figure 9: Increasing the strength of connections*

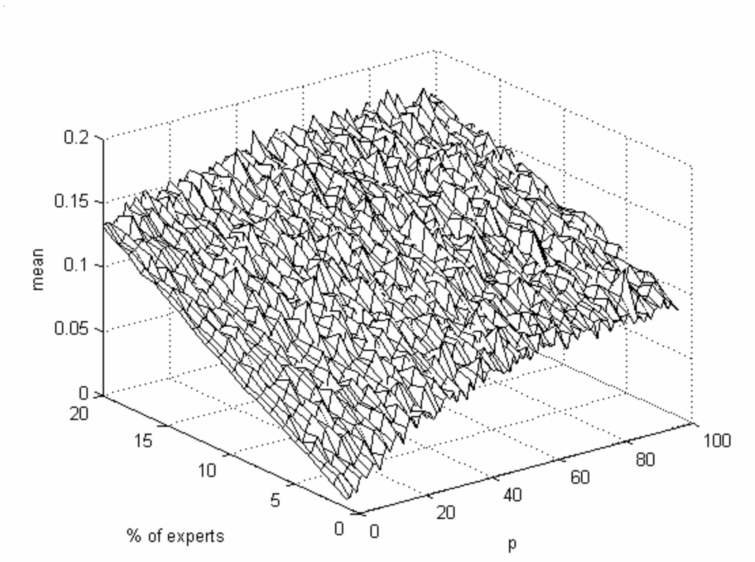



As with the previous sets of simulated results three percent of agents are experts. As illustrated in Figure 9 increasing the strength of the connections from these experts has very little effect on the level of volatility. Therefore any increase in volatility of expectations can only be coming from an increase in the number of connections from expert agents.

### 5.3 Biases in Agent Expectations

Inflation expectations are a biased predictor of actual inflation tending to over predict actual inflation in periods of low inflation and conversely under predict in periods of high inflation (Christensen 1996) (Dahl & Hansen 2001) (Makin 2003).[12] To see if agents in our model are also biased in the same direction simulations were run examining the mean level of agent's expectations against the number of experts and the level of randomness. The results for the low state of the world ($V = 0$) are depicted in Figure 10. Once you move away from an ordered network the mean level of expectations rises sharply reaching a plateau. As you increase the number of expert agents the mean continues to rise.

*Figure 10: Overestimating Bias*



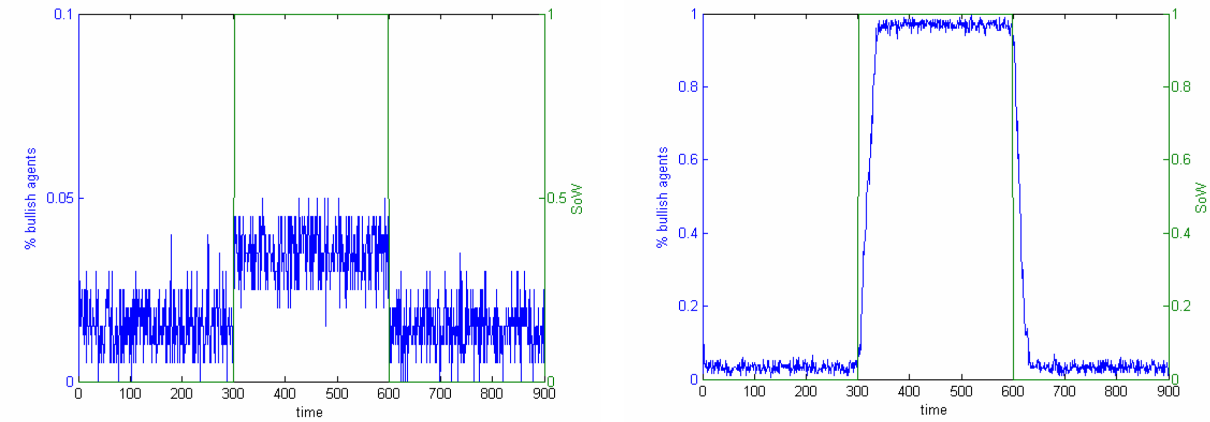

## 6 INFORMATION CASCADES AND BUBBLES

In this section we explore whether certain network structures can indeed lock agents into an information cascade which could potentially lead to the formation of bubbles. In the scenarios considered thus far herd behaviour increases the level of volatility in the market but does not lead to long run and significant misinterpretation of the state of the world. The number of connections between herd agents $k$ is increased from two to four. In this scenario all agent (both herd and expert) are connected to the two agents on either side of them. Five percent of all agents are expert agents. It is found that, where the network is ordered agents enter into an information cascade (see Figure 11a), however for $p \geq 1$ the cascade is broken and volatility decreases significantly (Figure 11b). When $k$ is greater

[12] Christensen (1996) examined Danish data on inflation expectations, Dahl & Hansen (2001) American and Makin (2003) Australian.

than four agents are always in an information cascade with a result similar to Figure 11a (not shown here).

*Figure 11: Five Expert Agents p = 0% then 5% and k = 4*




a b

It is therefore possible that, under certain network structures herd behaviour can lead to information cascades and potentially this locking into an expectation could lead to the formation of speculative bubbles. Intuitively, as long as the average number of connections between agents is low, information can flow within the social network. As the number of connections increase information flows become congested as the actions of other agents dominate their own private signal. The surprising result here is that the number of connections per agent does not need to be large before information becomes blocked.

Interestingly speculative bubbles in financial markets are characterised by excessive reporting in the media. It also dominates social discussions between neighbours or within the workplace. You also get the situation where people want to jump on to the bandwagon in fear of missing out on opportunities. At the same time the number of dissenting views seems to fall away. A recent example was the 2001 tech boom when most analysts were sprouting the view that technology stocks were the future and that the old pricing mythologies no longer applied. Views to the contrary were often put down as outdated and ignored - a position which appeared justified as those with the contrarian view performed poorly relative to the market.

## 7 CONCLUSION

Survey data on agent expectations appear to experience inertia, remaining relatively stable for protracted periods punctuated with the occasional structural shift initiated by exogenous changes (Flieth & Foster 2002). The data is also characterised with an underlying level of volatility punctuated with periods of higher volatility. In addition inflation expectations is a biased estimator of actual inflation tending to over predict during periods of low inflation and under predict during periods of high inflation

(Christensen 1996) (Dahl & Hansen 2001) (Makin 2003). It is found that social interaction and herd behaviour, within a small world network, displays similar characteristics to this dynamic process.

A multi-agent model is developed based on the social learning framework initially developed by BHW (1992) and Banerjee (1992). The social learning takes place in a network consistent with the work on small worlds by Watts (1999). Moving away from an ordered network steadily increases the level of volatility, before reaching a plateau. Assuming that all social networks have small world characteristics with $p$ > 10% then there is an inherent level of volatility in expectations formation. Increasing the number of expert agents does not have a significant impact (i.e. is not additive) on the underlying level of volatility. Increasing the number of connections from expert agents increases the level of volatility in agent expectations which is additive with the underlying level associated with the small world characteristics. Within such a framework periods of high volatility could occur during times of strong public debate and high levels of interest in the community.

Increasing the number of expert agents while not significantly effecting the level of volatility does increase (decrease) the mean level of expectations when the state of the world is low (high) (a low state of the world is seen as equivalent to low expectations and vis versa). Finally, changing the state of the world, through an exogenous process results in a structural shift in the level of agent expectations as the information that the state has changed filters from the experts to the herd agents' thorough contagion.

Herd behaviour is often cited as one of the forces behind speculative price bubbles and crashes in stock markets. It is found that under certain network structures, where the number of connections between agents is increased, herd behaviour will lead to information cascades potentially leading to the formation of speculative bubbles.

There are a number of potentially testable theories which arise from the work in this paper. Does volatility in agent expectations increase when communication from experts rises (particularly at times when the majority of experts appear to be singing from the same hymn book?) and do bubbles occur during times when the number of connections between agents are high?

There are also a number of extensions to the model. In particular it appears that the speed of convergence is very rapid even when only one percent of agents are experts. This is consistent with the work of Bala & Goyal (1998) and Gale & Kariv (2003). However, given the variability in the speed of convergence in actual data it appears that this model is only telling part of the story.

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