Measuring Inequality of Well-Being
A proposal based on a Multidimensional Gini Index

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Preliminary version

Abstract

Individual well-being is inherently a multidimensional concept. An attempt to measure inequality in well-being should take this multidimensionality explicitly into account. In this paper we follow a normative procedure to derive a measure of well-being inequality from its underlying multidimensional social evaluation function. The social evaluation function itself is characterized by an explicit two step approach. In a first step, an index of individual well-being is derived from a set of attractive properties. The second step aggregates the individual well-being indices into a measure of societal well-being. Hereby we allow the evaluation of well-being to depend both on its level and on the position of the individual in the total distribution. We investigate the role of a multidimensional version of the Pigou-Dalton transfer principle and the sensitivity of the social evaluation function to changes in the correlation between the dimensions. The obtained measure is illustrated using household data from Russia and Indonesia and on aggregated international data on three dimensions of well-being: material standard of living, health and education.

Keywords: objective well-being, multidimensional inequality, single parameter Gini, multidimensional transfer principle

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1 Introduction

Many argue that well-being is inherently a multidimensional concept (Rawls 1971, Sen 1985, Streeten 1994). Dimensions of individual well-being can be typical outcome variables such as per capita income, health status, educational outcome, or housing characteristics. These dimensions are neither freely tradable nor perfectly correlated to income. As a result, an attempt to measure inequality of well-being should take this multidimensionality explicitly into account. In this paper we present a multidimensional generalization of the Gini coefficient to measure inequality in well-being.

The Gini coefficient is probably the best known inequality measure in economics. From its introduction in the economics literature in 1912 by the Italian statistician Corrado Gini (Gini 1912, 1921), the coefficient has become increasingly popular as a tool to measure inequality.

The present paper uses a normative procedure to derive a measure of well-being inequality.[1] Starting from a set of attractive properties, we characterize a multidimensional social evaluation function, from which we derive a measure of inequality. This procedure has the advantage of bringing the chosen properties and value judgements explicitly to the fore, which in turn allows researchers to form a clear opinion on the attractiveness of the inequality measure for the problem at hand. The characterization of the social evaluation function is carried out in two distinct steps in order to make the multidimensional and complex characterization problem as tractable and intuitive as possible.

In a first step, we derive an *objective* well-being index that summarizes the outcomes of every individual across the dimensions of well-being. Contrary to the approaches based on *subjective* well-being or happiness, these well-being indices are *not* dependent on subjective individual preferences over the dimensions. Rather, the 'objective' well-being function reflects the preferences of the society (or external observer) over the different dimensions. The well-being function is thus common to all members of society and permits well-being judgements that are not purely subjective but interpersonally justifiable and comparable (Gaspar 1998).[2]

The second step aggregates the well-being indices across individuals to obtain a measure

[1] For a recent survey of multidimensional normative inequality measures, the reader is referred to Weymark (2006).

[2] A potential concern with the 'objective' well-being approach is that it is overly paternalistic or perfectionist, since it represents the preferences of the external observer about what constitutes a good life for the individuals, and not the preferences of the individuals themselves (Fleurbaey 2005). Fleurbaey and Trannoy (2003) prove the impossibility of combining differences in individual preferences with multidimensional egalitarianism. However, this critique is less prevalent when the preferences of the external observer are derived by a democratic process or when personal satisfaction or 'happiness' is included as a relevant dimension of well-being.

of the well-being of a society as a whole. An essential feature of this step is that we allow that the contribution of an individual's well-being to the total societal well-being depends on the position or rank of that individual in the total distribution. The result of the two-step approach is a multidimensional rank-dependent social evaluation function. We investigate the impact of imposing multidimensional distributional concerns. One concern is about the equality of the dimension-wise distributions and the other concern is about the correlation between the dimensions. From the obtained social evaluation function, a family of inequality measures flows rather naturally. The obtained family of inequality measures is a multidimensional generalization of the one-dimensional S-Gini inequality measure.

Our paper is related to the recent work of Gajdos and Weymark (2005) on normative multidimensional generalized Gini indices. Driven by different sets of properties, the authors obtain a multidimensional generalized Gini social evaluation function which turns out to be a weighted-average of the one-dimensional Gini indices of the different dimensions. The two-step approach of Gajdos and Weymark aggregates first across individuals and then across dimensions and can be considered to be the mirror-image of the procedure used here. We will argue that first aggregating across dimensions and then across individuals is more attractive, since it is more in line with the conceptual framework of welfare economics (Dutta, Pattanaik and Xu 2003) and the literature on multidimensional inequality (Maasoumi 1999). Moreover, the procedure used in this paper does not exclude *a-priori* sensitivity to the correlation between the dimensions. The importance of correlation between the dimensions in the analysis of multidimensional inequality is brought under attention by Atkinson and Bourguignon (1982), Rietveld (1990) and Tsui (1999).

Outside the normative approach, two other broad strategies have been followed to generalize the Gini coefficient into multiple dimensions, each extending an alternative one-dimensional definition of the Gini coefficient.[3] Koshevoy and Mosler (1996) introduced the *Lorenz zonoid* as m-dimensional generalization of the standard Lorenz curve. From the volume of the Lorenz zonoid, a multidimensional Gini coefficient can be derived naturally (Koshevoy and Mosler 1997). An alternative strategy is followed by Arnold (1987), Koshevoy and Mosler (1997) and Anderson (2004), who extend the definition based on the sum of all distances between pairs of individuals. In particular, they propose a multidimensional *distance measure* to measure the pair wise distances between the vectors of outcomes. It has the virtue of being easy to implement, but the essential

[3] For an overview of the different interpretations of the one-dimensional Gini coefficient, see Sen (1973), Anand (1983) or Yitzhaki (1998). Arnold (2005) gives a survey of the recent developments on multidimensional generalizations of the Gini coefficient.

underlying value judgements are made implicit, which makes the index less attractive from a normative viewpoint.

The rest of the paper is structured as follows. Section 2 derives an individual well-being index that aggregates individual's outcomes across all dimensions of well-being. Section 3 characterizes the single parameter Gini social evaluation function to aggregate the well-being indices across individuals. Distributional concerns are introduced in section 4, paying special attention to a multidimensional generalization of the one-dimensional Pigou-Dalton transfer principle and the effect of changes in correlation between dimensions. From the obtained social evaluation function a relative multidimensional single parameter Gini inequality measure is derived in section 5. Section 6 illustrates the use of the obtained measure using household data from Russia and Indonesia and on aggregated international data. The exercise shows that the obtained trend in well-being inequality is sensitive to the normative choices made in the construction of the underlying social evaluation function. Specifically, we find that the sequence in which the aggregation between dimensions and individuals is carried out plays an important role in the empirical results. Section 7 concludes the paper.

2 Aggregation across Dimensions of Well-Being

Let $\mathcal{N} = \{1, ..., n\}$ be the set of individuals i and $\mathcal{M} = \{1, ..., m\}$ the set of dimensions j. A distribution matrix X is an $n \times m$ strictly positive real valued matrix whose element x_i^j represents the outcomes of individual i on dimension j. When m =1, matrix X is a one-dimensional vector. The domain of the distribution matrices is denoted $\mathcal{D}$ and is restricted to the set of strictly positive real-valued distribution matrices.

Define x_i as the row vector of distribution matrix X that represents the outcomes of individual i and x^j the column vector of the same matrix, representing the distribution of the j-th dimension of well-being. The set $\mathcal{V}$ is the set of admissible outcome vectors x_i. The *well-being relation* $\succeq$ is a binary relation on $\mathcal{V}$. It captures the preferences of the external observer with respect to bundles of outcomes. By $x_i \sim y_i$ we denote that the vector of outcomes x_i is socially indifferent to vector y_i. Strict social preference for x_i will be denoted $x_i \succ y_i$.

In this section we characterize a *well-being function* $S : \mathcal{V} \to \mathbb{R}$, which represents $\succeq$. The well-being function aggregates outcomes across the multiple dimensions of well-being and captures different value judgements regarding trade-offs between dimensions and admissible transformations, among others.[4] We crystallize those value judgements by presenting them as a set of explicit properties. We do not claim that the set of properties laid down here is the only one possible, *au-contraire*, but we suggest that it represents an attractive set for the problem of measuring well-being. The problem of finding a well-being function that satisfies a set of properties is similar to the standard problem of aggregating individuals' preferences into a social welfare function, hence we can make use of existing social choice results.[5] The first three properties are standard ones.

Property 1. ***Ordering (ORD$_{\succeq}$)*** *The binary relation $\succeq$ is reflexive, complete and transitive on $\mathcal{V}$.*

Property 2. ***Continuity (CONT$_{\succeq}$)*** *The sets $\{y_i \in \mathcal{V} \mid y_i \succ x_i\}$ and $\{y_i \in \mathcal{V} \mid x_i \succ y_i\}$ are open for all x_i in $\mathcal{V}$.*

Property 3. ***Monotonicity (MON$_{\succeq}$)*** *For all x_i, y_i in $\mathcal{V}$, if $x_i \neq y_i$ and $x_i^j \geq y_i^j$ for all dimensions j in $\mathcal{M}$, then $x_i \succ y_i$.*

The first property imposes some structure on the well-being relation $\succeq$, requiring it to be a complete preorder without cycles. The second property ensures that the well-being relation is continuous and, hence, not oversensitive to minor changes in the outcomes, for example caused by measurement errors. Monotonicity captures the intuition that all dimensions in $\mathcal{M}$ are desirable. Regardless of the exact form of the preferences of the external observer, this third property requires that increasing the outcome in any dimension, without decreasing an outcome in any other dimension, leads to a more preferable outcome vector.

The next property introduces separability across dimensions. More specifically, when two outcome vectors display the same outcome for a certain dimension, the exact level of the common outcome is not decisive to the well-being relation. An example helps to clarify: suppose two individuals have the same outcome in the income dimension and different outcome levels in health and education, then we assert that the exact level of income is not important to order the individuals with respect to their well-being. This property imposes a separable structure to the well-being function S and excludes, for instance, rank-dependent well-being functions where the contribution of a dimension depends on

[4] We do not discuss which dimensions of well-being should be included, rather we assume that these are either obtained by a democratic process or given by philosophical reasoning like the primary goods defined by Rawls (1971), the list of 'functionings' proposed by Nussbaum (2000), or the basic needs approach advocated by Streeten (1994).

[5] Also Tsui (1996) and Ebert and Welsch (2004) exploit the similarity with the social choice problem to derive improvement indices and meaningful environmental indices, respectively.

its ranking in the outcome vector of the individual. Let $\mathcal{J}$ be a subset of $\mathcal{M}$, the set of dimensions.

Property 4. ***Dimension Separability (DSEP$_\succeq$)*** *For all $x_i, y_i, \widetilde{x}_i, \widetilde{y}_i$ in $\mathcal{V}$, the relation $\succeq$ is dimension separable if $x_i \succeq y_i \Leftrightarrow \widetilde{x}_i \succeq \widetilde{y}_i$, if for all $j \in \mathcal{J}, x_i^j = y_i^j$ and $\widetilde{x}_i^j = \widetilde{y}_i^j$, whereas for all $j \in \mathcal{M} \backslash \mathcal{J}, x_i^j = \widetilde{x}_i^j$ and $y_i^j = \widetilde{y}_i^j$.*

The fifth property, homotheticity, asserts that the dimensions of an outcome vector can be rescaled without changing the well-being relation.[6] We consider two versions of this property. First, strong homotheticity allows the rescaling of every dimension with a dimension-specific rescaling factor without affecting the well-being order (Tsui 1995).[7] When imposing strong homotheticity, the ordering of well-being can be obtained irrespective of the measurement units employed. For instance, one can order two outcome vectors, remaining completely agnostic about whether the income dimension is measured in dollars, dollarcents or euros. Weymark (2006) advocates the use of this property as the appropriate invariance property when the dimensions of well-being are different in nature, such as income expressed in dollars and life expectancy in years. However, strong homotheticity may seem to be too strong a requirement, since the measurement units used for each dimension are often known and can be used as information when making comparisons between outcome vectors. Therefore, a weaker version of the property is also proposed. Weak homotheticity allows a rescaling of all dimensions with the same proportional change. Bourguignon (1999) argues that weak homotheticity is more appropriate when the dimensions are naturally measured in comparable units.[8] Formally, we state:

Property 5. ***Strong Homotheticity (SHOM$_\succeq$)*** *For all x_i, y_i in $\mathcal{V}$, and for all positive $m \times m$ diagonal matrices Λ, $x_i \succeq y_i \Leftrightarrow x_i\Lambda \succeq y_i\Lambda$.*

Property 6. ***Weak Homotheticity (WHOM$_\succeq$)*** *For all x_i, y_i in $\mathcal{V}$, and for all $\lambda > 0$, $x_i \succeq y_i \Leftrightarrow x_i\lambda \succeq y_i\lambda$.*

The following proposition brings together the above properties, and derives the sole well-being function that satisfies them all.

[6] Homotheticity is a specific case of the invariance axioms introduced in the social choice literature by authors like Sen (1970). In social choice theory, homotheticity is commonly referred to as ratio scale measurability. We refer the reader to Bossert and Weymark (2000); d'Aspremont and Gevers (2002) or Roberts (2005) for recent surveys.

[7] Note that an ordering that satisfies SHOM$_\succeq$ also satisfies DSEP$_\succeq$.

[8] Examples include aggregating incomes at different periods of time (intertemporal analysis and chronic poverty) or from different sources (earnings, transfers, profits, etc).

Proposition 1. *A well-being relation $\succeq$ on $\mathcal{V}$ satisfies*
(a) ORD$_{\succeq}$, CONT$_{\succeq}$, MON$_{\succeq}$, SHOM$_{\succeq}$, if and only if $\succeq$ can be represented by a well-being function S, ordinally equivalent to:

$$S(x_i) = \prod_{j=1}^{m} (x_i^j)^{w_j}, \tag{1}$$

with $w_j > 0$ for all $j \in \mathcal{M}$, and $\sum_{j=1}^{m} w_j = 1$,

(b) ORD$_{\succeq}$, CONT$_{\succeq}$, MON$_{\succeq}$, DSEP$_{\succeq}$, WHOM$_{\succeq}$, if and only if $\succeq$ can be represented by a well-being function S, ordinally equivalent to:

$$S(x_i) = \left(\sum_{j=1}^{m} w_j (x_i^j)^{\beta} \right)^{(1/\beta)}, \tag{2}$$

with $w_j > 0$ for all $j \in \mathcal{M}$, and $\sum_{j=1}^{m} w_j = 1$.

Proof. For (a) see Tsui and Weymark (1997; Theorem 4), and for (b) Blackorby and Donaldson (1982; Theorem 2). □

The result of proposition 1(a) is, in some respects, disappointing and reveals how restrictive the requirement of SHOM$_{\succeq}$ is. In the presence of the other properties, choosing for SHOM$_{\succeq}$ leads inevitably to a Cobb-Douglas well-being function which has unit elasticity of substitution, i.e. $\sigma = 1$. Other elasticities of substitution between the dimensions cannot be obtained without relaxing one of the properties (Tsui and Weymark, 1997). A possible relaxation is to impose WHOM$_{\succeq}$ instead of SHOM$_{\succeq}$ which leads, in combination with DSEP$_S$ and the other properties, to the constant elasticity of substitution (CES) well-being function of proposition 1(b). Parameter β in expression (2) reflects the degree of substitutability between the dimensions of well-being. In particular, β is related to the elasticity of substitution between the dimensions, and equals $1 - 1/\sigma$. When $\beta = 0$, the limit case leads to expression (1). When $\beta = 1$ the dimensions of well-being are seen as perfect substitutes. As β tends to $-\infty$, dimensions tend to perfect complementarity; at the extreme, individuals are judged upon their worst outcome.[9]

The positive weights w_j sum to 1 and reflect the relative importance of the different dimensions. In interplay with parameter β, the weights determine the trade-offs between the dimensions (Decancq and Lugo 2008).

[9] This extreme case is excluded by the monotonicity property, and should be considered as a limiting case.

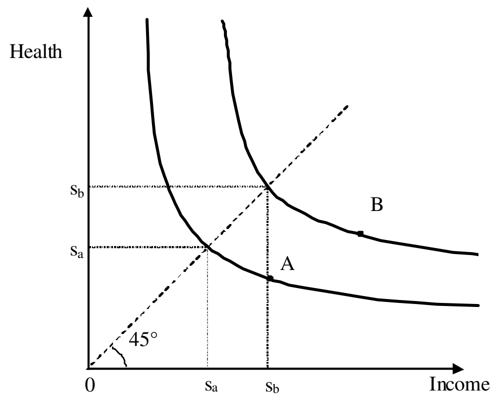

Figure 1: The equally distributed equivalent outcome in the two-dimensional income-health space.

Proposition 1 characterizes a broad class of well-being functions that are ordinally equivalent to expression (1) or (2). In other words, the above proposition specifies the m-dimensional iso-well-being curves,[10] but not the cardinal labels of these curves. For the purpose of measuring inequality in well-being, however, we are interested precisely in the inequality between those cardinal labels. The cardinalization is hence a last indispensable and delicate step.

One convenient cardinalization of the ordinally specified functions, is the *equally distributed equivalent outcome*, denoted s_i. The equally distributed equivalent outcome is defined analogously to the equally distributed income in one-dimensional inequality measurement (Kolm 1969, Atkinson 1970). It is the outcome level s_i that obtained equally by individual i in all dimensions, leads to the same level of well-being as the observed outcome vector, so that $S(s_i, ..., s_i) = S(x_i)$. For the well-being functions in expression (1) or (2), $s_i = S(x_i)$. Figure 1 depicts the iso-well-being curves for two individuals –A and B– and their equally distributed equivalent outcomes s_a and s_b in the two-dimensional income-health space.

To ease comparisons, one might prefer that all well-being levels are in a common interval, for instance $[0, 1]$. This can be obtained by taking a linear transformation of the well-being measures s_i. Consider, for instance the linearly transformed index s'_i,

$$s'_i = \frac{s_i - s_{\min}}{s_{\max} - s_{\min}}, \tag{3}$$

where $s_{\min}$ denotes the lowest level of well-being in the society and $s_{\max}$ the highest.

[10] A iso-well-being curve, is an m-dimensional hyperplane that connects the m-dimensional attainment vectors between which the external observer is indifferent.

In the next section we impose a linear invariance property at the level of the well-being levels. In particular, we assert that a common rescaling or translation of the well-being levels will not affect the social evaluation ordering of two societies.

The well-being function characterized by proposition 1 and cardinalized by its equally distributed equivalent outcome is a popular measure of well-being in the literature. The Human Development Index advocated by the UNDP, for instance, is a special case of expression (2), with $\beta = 1$, and weights w_j equal to 1/3. Other examples can be found in the literature on multidimensional inequality measurement. Maasoumi (1986, 1999) derives a CES well-being function based on different considerations rooted in information theory.

3 Aggregation across Individuals

The $n \times 1$ *well-being vector* $S_X = [S(x_{[1]}), S(x_{[2]}), ..., S(x_{[n]})]'$ consists of the well-being measures for the n individuals, obtained from the n rows of distribution matrix X. Well-being vectors are ordered from the highest well-being level to the lowest such that $S(x_{[1]}) \geq S(x_{[2]}) \geq ... \geq S(x_{[n]})$. Let $\mathcal{S}$ denote the set of the (ordered) well-being vectors. The social evaluation relation R is a binary relation on $\mathcal{S}$. The social evaluation relation will be referred to as R^n when it is advantageous to make the dimensions of the well-being vectors ordered by the relation explicit. By $S_X \ \mathsf{P} \ S_Y$ we denote that the vector of well-being indices S_X is strictly socially preferred to S_Y. Social indifference between S_X and S_Y will be denoted $S_X \ \mathsf{I} \ S_Y$. The *social evaluation function* $W : \mathcal{S} \rightarrow \mathbb{R}$ is a real-valued function that represents the relation R.

In this section we characterize a class of social evaluation functions W that aggregate the well-being indices across individuals. We follow a procedure that is standard in the literature on one-dimensional inequality measurement and is similar to the one of the previous section. However, because the nature of the problem of aggregation across individuals differs from that of aggregating across dimensions, the set of attractive properties and the resulting aggregation function are slightly different.

The first three properties are similar to their namesakes in the previous section. Also the interpretation is analogous, so we restrict ourselves to a summary treatment.

Property 7. ***Ordering (ORD$_\mathsf{R}$)*** *The binary relation* R *is reflexive, complete and transitive on* $\mathcal{S}$.

Property 8. ***Continuity (CONT$_\mathsf{R}$)*** *The sets* $\{S_Y \in \mathcal{S} \mid S_Y \mathsf{P} S_X\}$ *and* $\{S_Y \in \mathcal{S} \mid S_X \mathsf{P} S_Y\}$ *are open for all* S_X *in* $\mathcal{S}$*.*

Property 9. ***Monotonicity (MON$_\mathsf{R}$)*** *For all* S_X*,* S_Y *in* $\mathcal{S}$*, if* $S_X \neq S_Y$ *and* $S(x_{[i]}) \geq S(y_{[i]})$ *for all individuals* $i \in \mathcal{N}$*, then* $S_X \mathsf{~P~} S_Y$*.*

The fourth property imposes individual separability on the social evaluation of the ordered well-being vectors. In other words, the comparison of two ordered well-being vectors is not affected by the magnitude of equal well-being indices in both well-being vectors *as long as the initial ranking is maintained.*[11] Let $\mathcal{I}$ be a subset of $\mathcal{N}$, the set of individuals.

Property 10. ***Individual Separability (ISEP$_\mathsf{R}$)*** *For all* $S_X, S_Y, S_{\widetilde{X}}, S_{\widetilde{Y}}$ *in* $\mathcal{S}$*, the relation* R *is individual-separable if* $S_X \mathsf{~R~} S_Y \Leftrightarrow S_{\widetilde{X}} \mathsf{~R~} S_{\widetilde{Y}}$*, if for all* $i \in \mathcal{I}$*,* $S(x_{[i]}) = S(y_{[i]})$ *and* $S(\widetilde{x_{[i]}}) = S(\widetilde{y_{[i]}})$*, whereas for all* $i \in \mathcal{N} \backslash \mathcal{I}$*,* $S(x_{[i]}) = S(\widetilde{x_{[i]}})$ *and* $S(y_{[i]}) = S(\widetilde{y_{[i]}})$*.*

As noted at the end of the previous section, it might be useful to use a linear transformation of the well-being levels, for reasons of comparison. By imposing the next property we assert that the social evaluation relation is invariant to such common linear transformations of the well-being levels.

Property 11. ***Linear Invariance (LINV$_\mathsf{R}$)*** *For all* S_X*,* S_Y *in* $\mathcal{S}$*, and for all* κ, λ *with* $\lambda > 0$*,* $S_X \mathsf{~R~} S_Y \Leftrightarrow S_X\lambda + \kappa 1_n \mathsf{~R~} S_Y\lambda + \kappa 1_n$*.*

The following proposition derives the sole social evaluation function that satisfies all the above properties.

Proposition 2. *A social evaluation relation* R^n *on* $\mathcal{S}$ *satisfies ORD$_\mathsf{R}$, CONT$_\mathsf{R}$, MON$_\mathsf{R}$, ISEP$_\mathsf{R}$, LINV$_\mathsf{R}$, if and only if* R^n *can be represented by a social evaluation function W, ordinally equivalent to:*

$$W(S_X) = \sum_{i=1}^{n} a_i S(x_{[i]}), \tag{4}$$

with $a_i > 0$ *for all* i *in* $\mathcal{N}$*, and* $\sum_{i=1}^{n} a_i = 1$*.*

Proof. See Ebert (1988; Corollary 5). □

[11] Ebert (1988) introduced separability restricted on the ordered domain in the literature on inequality measurement and compares it to the much stronger unrestricted separability property.

From expression (4) it is clear that the case with only one dimension of well-being $m = 1$, collapses to the standard one-dimensional generalized Gini social evaluation function (Weymark 1981).

The $n \times 1$ vector of non-negative welfare weights a_i contains the weights attached to the individuals in the aggregation across individuals. These weights depend on the rank of the individual in the distribution of all well-being levels in the society. In other words, the contribution of the individual i to total well-being of the society depends on her individual well-being *level* $S(x_i)$ and her *rank* in the distribution through the weight a_i, which makes the social evaluation function in expression 4, particularly interesting. In recent surveys on subjective well-being it has been documented that one's evaluation of her well-being depends crucially on both the level and the relative position *vis-à-vis* other individuals (Ferrer-i-Carbonell 2005, Luttmer 2005).[12] By choosing the weights a_i we can reflect different ethical viewpoints the external observer might hold regarding the aggregation across individuals. Two extreme cases are of interest: when $a_i = 1/n$ everybody's well-being contributes equally to the social evaluation function which becomes a simple average of the well-being indices of the individuals, consistent with the utilitarian perspective; on the other hand, when $a_n = 1$ and all other weights are zero the external observer considers solely the situation of the worst-off individual, which reflects a Rawlsian perspective.[13] An intermediate approach is, for example, obtained by considering $a_i = (2i-1)/n^2$, which makes W equivalent to a Gini social evaluation function.

The following two properties allow us to order the well-being vectors of different length. The principle of population states that if two vectors of well-being of population size n are equivalent, then the m-times replicated vectors are equivalent as well (Dalton 1920). In other words, the evaluation of the well-being vectors depends on their distribution functions, independent of the population size. Let $S_Y^{(p)}$ and $S_X^{(p)}$ be the p-times replicated vectors of S_Y and S_X.

Property 12. ***Principle of Population (POP_{R})*** *For all S_X, S_Y in $\mathcal{S}$, such that S_Y I^n S_X and any $m \geq 2$, we have that $S_Y^{(p)}$ $\mathsf{I}^{(n \cdot p)}$ $S_X^{(p)}$.*

Donaldson and Weymark (1980) introduce a second property to allow comparisons between populations with different sizes. Restricted aggregation asserts that the well-being of the k better-off individuals can be replaced by their equally distributed equivalent well-being, v_k, that is, the average well-being among these k individuals. The procedure

[12] A central finding of these surveys is that 'lagging behind the Joneses' diminishes well-being.

[13] The Rawlsian case is, strictly speaking, excluded by the monotonicity property, and should be considered as a limiting case.

of substituting the actual well-being indices by the respective equally distributed equivalent is a procedure of aggregation. The aggregation is restricted to the subgroup of the k better-off individuals, and therefore its name.

Property 13. ***Restricted Aggregation (RA_{R})*** *For all S_X in $\mathcal{S}$, and all $k \leq n$ we have $S_X \underline{\mathsf{I}}^n \, [v_k, ..., v_k, S(x_{[k+1]}), ..., S(x_{[n]})]$.*

Donaldson and Weymark (1980) prove the following result, leading to the *single parameter* Gini (S-Gini) social evaluation function.

Proposition 3. *A social evaluation relation R^n on $\mathcal{S}$ satisfies ORD_{R}, $CONT_{\mathsf{R}}$, MON_{R}, $ISEP_{\mathsf{R}}$, $LINV_{\mathsf{R}}$, POP_{R} and RA_{R} if and only if R^n can be represented by a social evaluation function W, ordinally equivalent to:*

$$W(S_X|\delta) = \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^{\delta} - \left(\frac{i-1}{n}\right)^{\delta} \right] S(x_{[i]}), \tag{5}$$

with $\delta > 0$.

Proof. See Donaldson and Weymark (1980; Theorem 2). □

The obtained class of social evaluation functions is a convenient one, since it allows the sequence of weights and the associated value judgements to be captured by a single parameter δ. Its one-dimensional member is the underlying social evaluation function of the popular class of *S-Gini inequality indices* (Donaldson and Weymark 1980, Kakwani 1980, Yitzhaki 1983). Parameter δ captures the bottom-sensitivity of the social evaluation function. If parameter δ equals 1, the social evaluation function is the unweighted average of the well-being indices of the individuals in the utilitarian tradition. The higher δ, the more weight is given to the bottom of the distribution, with as limiting case $\delta = +\infty$, which leads to a Rawlsian social evaluation function. If δ is smaller than 1, more weight is given to the best-off individuals. The standard Gini social evaluation function is obtained by setting $\delta = 2$.

4 Distributional concerns

Let $\underline{\mathsf{R}}_{\succeq}$ denote the *compound social evaluation relation* defined on the set of distribution matrices $\mathcal{D}$. We write $X \, \underline{\mathsf{R}}_{\succeq} \, Y$, to denote that distribution matrix X is socially preferred to Y according to all two step procedures that first order the outcome vectors according to $\succeq$, and then the obtained well-being vectors according to R. The *compound social evaluation function* $\underline{W} : \mathcal{D} \to \mathbb{R}$ is a real-valued function that represents the relation $\underline{\mathsf{R}}_{\succeq}$.

Bringing together the results of the previous two sections, the compound multidimensional social evaluation relation that satisfies the set of properties of proposition 1b on $\succeq$ and the set of properties of proposition 3 on R can be represented by a compound social evaluation function $\underline{W}$, which is ordinally equivalent to the following expression:

$$\underline{W}(X|\delta,\beta,w) = \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^{\delta} - \left(\frac{i-1}{n}\right)^{\delta} \right] \left(\sum_{j=1}^{m} w_j (x^j_{\{i\}})^{\beta} \right)^{(1/\beta)}, \tag{6}$$

where $\{i\}$ denotes the position of the individual in the distribution of well-being indices (and not the position in dimension j). In this expression, three parameters have to be specified. Parameter δ specifies the bottom sensitivity, or the weight given to worst-off individuals in the aggregation across the individuals, β captures the degree of substitutability between the dimensions of well-being and vector w collects the m weights w_js reflecting the relative importance attached to outcomes in the aggregation across dimensions.

The reader will note that so far we have not introduced any property that captures distributional concerns. In the standard one-dimensional analyses, distributional sensitivity is obtained by imposing some form of the *Pigou-Dalton* transfer principle. The principle states that a transfer of income from a poorer to a richer individual leads to a decrease in social welfare. Some proposals have been made to generalize the one-dimensional Pigou-Dalton principle to the multidimensional setting.[14] In this section we focus on a popular generalization and investigate the effect of imposing it within the multidimensional framework laid down in the previous sections and captured by expression (6). We also introduce a distributional property which is specific to the multidimensional setting related to the correlation between the dimensions. These two distributional concerns are defined on the space of the distribution matrices and take full account of the multidimensionality of the problem.[15]

[14] Examples of multidimensional generalizations of the Pigou-Dalton principle can be found in Kolm (1977) or more recently in Weymark (2006). These generalizations are rooted in the theory of multidimensional majorization (Marshall and Olkin, 1979; Chapter 15).

[15] An alternative approach would be to impose the Pigou-Dalton principle in the space of the well-being vectors, as it is done implicitly by Maasoumi (1986, 1999). Such an approach reduces the multidimensional problem to a one-dimensional one in the space of the well-being indices. It is a standard result from the one-dimensional inequality literature, that such a one-dimensional Pigou-Dalton principle will restrict the δ parameter to be not smaller than 1 (Ebert, 1988).

The first property asserts that the same mean-preserving transfer is carried out across dimensions such that dispersion is reduced in all of them, then the resulting distribution matrix is socially preferred to the original one.[16]

Property 14. ***Uniform Majorization (UM)*** *For all $X, Y \in \mathcal{D}$, and all $n \times n$ bistochastic matrices B, if $Y = BX$, then $Y \ \underline{\mathsf{R}}_{\succeq} \ X$.*

The property UM imposes restrictions on the parameters of the compound social evaluation function $\underline{W}$.

Proposition 4. *A compound social evaluation relation $\underline{\mathsf{R}}_{\succeq}$ satisfying the properties specified in Propositions 1b and 3 on $\succeq$ and R respectively, also satisfies UM if the representation of $\underline{\mathsf{R}}_{\succeq}$ satisfies $\beta < 1$ and $\delta > 1$.*

Proof. See Appendix A. Atkinson and Bourguignon (1982) point to another distributional

concern. They argue that a social evaluation should be sensitive to possible correlation between the dimensions. Tsui (1999) formalized this notion of correlation between the dimensions into the normative framework, by defining a correlation increasing transfer.

Definition 1. ***Correlation Increasing Transfer (CIT)*** *For all $X, Y \in \mathcal{D}$, Y is obtained from X through a CIT if $X \neq Y$, X is not a permutation of Y, and there are at least two individuals k and l such that, (i) $y_k = max\{x_k^j, x_l^j\}$ for all dimensions j, (ii) $y_l = min\{x_k^j, x_l^j\}$ for all dimensions j and (iii) $y_i = x_i$ for all $i \notin \{k, l\}$.*

The next property incorporates the idea that a distribution matrix Y that is obtained from X by a finite series of correlation increasing transfers, should be socially inferior (Tsui, 1999). If two distribution matrices have identical marginal distributions, the one with lower correlation between the dimensions is preferred. The property captures the idea of compensating inequalities among different dimensions, hence implicitly assuming that dimensions are substitutes.[17]

Property 15. ***Correlation Increasing Majorization (CIM)*** *For all $X, Y \in \mathcal{D}$, if Y is obtained from X by a finite series of correlation increasing transfers , then $X \ \underline{\mathsf{R}}_{\succeq}$ Y.*

[16] Although this generalization of the one-dimensional Pigou-Dalton transfer principle seems to be the one most often used in the literature on multidimensional inequality (Tsui, 1995, Tsui 1999, Weymark, 2006), it is not uncontroversial. Dardanoni even calls it "uninformative for evaluating the amount of inequality in society" (1995). Indeed, uniform majorization is a strong condition by requiring that the same mean preserving decrease in dispersion is applied in every dimension of well-being.

[17] Bourguignon and Chakravarty (2003) suggest that depending on the nature of the dimensions, the opposite property could be considered.

We investigate the introduction of the correlation increasing majorization criterion within the multidimensional S-Gini framework developed in the previous sections.

Proposition 5. *A compound social evaluation relation* $\underline{\mathsf{R}}_{\succeq}$ *satisfying the properties specified in Propositions 1b and 3 on* $\succeq$ *and* R *respectively, cannot satisfy CIM.*

Proof. See Appendix A.

A formal proof of this proposition is left to the Appendix. Intuitively, as a result of a CIT invariably at least one person has gained in terms of well-being and another has lost. Moreover, after the transfer the looser has, by monotonicity of the well-being relation, a lower well-being level than the winner. Whether the losses of the looser outweigh the gains of the winner in the eyes of the external observer depends on his bottom sensitivity. To make sure that for *all possible* initial distribution matrices X societal well-being decreases after a sequence of CIT's, one has to opt for an extreme bottom sensitive weighting scheme that gives all weight to the worst-off, which is excluded by monotonicity of the social evaluation relation.

On the other hand, for a *given* initial distribution matrix X_1 and any matrix Y_1 obtained by a sequence of CIT's from X_1, there exists a minimal bottom sensitivity δ_1, such that for all δ it holds that $\delta_1 \leq \delta$ the losses of the looser outweigh the gains of the winner, and thus $\underline{W}(Y_1|\delta, \beta, w) \leq \underline{W}(X_1|\delta, \beta, w)$. In practice, the value of δ_1 for a given X_1 (and β and w_j's) is hard to obtain since the losses have to outweigh the gains for all possible sequences of CIT's from the initial distribution matrix X_1.

Of particular interest is the specific sequence of CIT's of X_1 that leads to a perfectly correlated distribution matrix Y_1^*, in which individual 1 gets the highest outcome in all m dimensions, the second individual gets the second highest outcomes and so on until the n-th individual is bottom-ranked in all dimensions.[18] Dardanoni (1995) refers to that sequence of CIT's as the *unfair rearrangement*, and argues that an unfair rearrangement should lead to a decrease in welfare. By calculation, the minimal bottom sensitivity δ_1^* can be obtained such that for all $\delta_1^* \leq \delta$, an unfair rearrangement of a given distribution matrix X_1 leads to a decrease in societal well-being: $\underline{W}(Y_1^*|\delta, \beta, w) \leq \underline{W}(X_1|\delta, \beta, w)$.[19]

In sum, to be sure that the obtained compound social evaluation function satisfies at

[18] Consider the following sequence of CIT's. First a CIT is carried out between individual 1 and the m individuals having the highest attainment in each of the m dimensions. Then carry out a CIT between individual 2 and the m individuals with the second highest attainment in all dimensions. And so on. After maximal $(n-1)m$ CIT's Y' is obtained.

[19] Note that $\delta_1^* \leq \delta_1$. Assume otherwise, for the δs in the open interval (δ_1, δ_1^*) the sequence of CIT's called an 'unfair rearrangement' would lead to increase in societal well-being (since $\delta < \delta_1^*$), which contradicts that for all $\delta > \delta_1$ all sequences of CIT's lead to decrease in societal well-being.

least Dardanoni's notion that Y_1^* obtained by an unfair rearrangement of a given X_1 leads to a decrease of societal well-being, one has to selects a δ such that $\delta_1^* \leq \delta$. For a social evaluation function that decreases after every sequence of CITs of a given X_1, one selects δ so that $\delta_1 \leq \delta$. Finally, to be certain that the social evaluation function decreases after every sequence of CIT's of any initial distribution matrix X, (that is, to satisfy CIM) the external observer has to take an extreme bottom sensitive position $\delta = \infty$, which is contradicted by monotonicity of the social evaluation relation.

5 A Measure of Well-being Inequality

In the one-dimensional normative approach, a relative inequality measure is derived from its underlying social evaluation function as the fraction of total societal well-being wasted due to inequality (Atkinson 1970, Kolm 1969, Sen 1973). In a seminal article, Kolm (1977) generalizes the one-dimensional definition in the multidimensional setting as the fraction of the aggregate amount of each dimension of a given distribution matrix that could be destroyed if every dimension of the matrix is equalized while keeping the resulting matrix socially indifferent to the original matrix according to $\underline{\mathsf{R}}_{\succeq}$ (see also Weymark 2006). Formally, a relative inequality measure $I_R(X)$ is defined as the scalar that solves

$$[1 - I_R(X)]X_\mu \;\; \underline{\mathsf{I}}_{\succeq} \;\; X \; , \tag{7}$$

where X_μ is the equalized distribution matrix defined such that the all the elements in the jth column of the matrix are the dimension-wise mean $\mu(x^j)$. Applying the compound social evaluation function obtained in (6) the following multidimensional S-Gini inequality index can be obtained,

$$I_R(X) = 1 - \frac{\sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^{\delta} - \left(\frac{i-1}{n}\right)^{\delta} \right] \left(\sum_{j=1}^{m} w_j (x_{\{i\}}^j)^{\beta} \right)^{1/\beta}}{\left(\sum_{j=1}^{m} w_j \mu(x^j)^{\beta} \right)^{1/\beta}}, \tag{8}$$

where $\beta < 1$ and $\delta > 1$.

6 Empirical examples

In this section we give three examples how the obtained measures can be applied to real world datasets. The first two examples are based on household data from Russia and Indonesia respectively, the latter on aggregated international data. In all examples we consider multidimensional well-being consisting of three dimensions: material standard of living, health and education.

6.1 Russian Well-being Inequality between 1995 and 2003

As a first example, we consider the question whether the Russian society is more equal in 2003 than it used to be in 1995. During this decade the Russian Federation underwent a far-going transition from a centrally planned economy to a free market economy. Moreover, Russia was hit by a severe financial crisis in August 1998. Many studies have documented that income inequality increased over this period (See, for instance, Brainerd 1998 or Ferreira 1999). Others have studied the dramatic decrease in health status and life expectancy in the late 90s (Amongst others Moser, *et al.* 2005). It is therefore interesting to see the evolution of well-being inequality over this decade when including in addition to material standard of living also other dimensions such as health and education in the analysis.

The sample comes from the Russian Longitudinal Monitoring Surveys (RLMS), a series of nearly annual, nationally representative surveys designed to monitor the effects of Russian reforms on health and economic welfare. We use three indicators for the respective dimensions of well-being: equivalent real household expenditures, a constructed health indicator and years of schooling. First, equivalent real household expenditures are widely used in the literature as indicator of material standard of living. We use the square root of household size as equivalence scale. For the health dimension, the RLMS is particularly rich on objective health indicators. We aggregate these indicators to come to one comprehensive measure of the health status of every individual. The weights in this aggregation are obtained from the regression coefficients in an estimation with self-reported health status as the dependent variable and a wide series of objective health indicators as explanatory variables.[20] By this procedure, the constructed measure is as close as possible to the self-reported health status of the individual while making sure that individuals with the same objective characteristics obtain the same health score. (For a similar procedure, see Van Doorslaer and Jones, 2003 or Nilsson, 2007). Third, years of schooling is constructed by aggregating the highest obtained grade and an indicator whether the individual followed higher education.

[20] Self-reported health is measured on a 5-point scale. We simplify the analysis by treating the ordinal variable as cardinal. Explanatory variables used in the OLS regression are indicators of diabetes, hart attack, anaemia or other health problems; indicators of a recent medical check-up, hospitalization or operation; life-style indicators such as smoking, regular exercises or jogging and age and gender dummies. All variables are highly significant and have the expected sign. The results can be found in Appendix B (table 6).

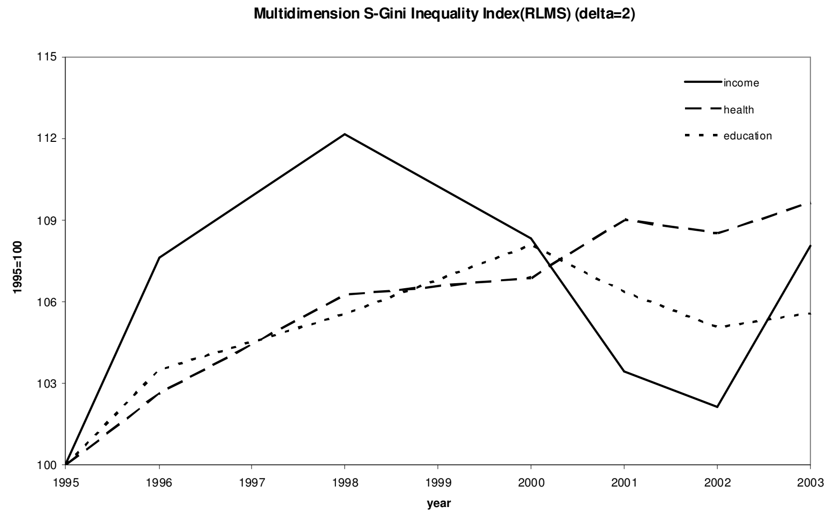

Figure 2: The evolution of Russian inequality measured by a dimension-wise S-Gini inequality index (for $\delta = 2$).

We only use the adult individuals for which we have data available for all three dimensions, leaving us with a sample of about 6,000 individuals in every wave (see row 1 of Table 1). Table 1 shows the average values of the three dimensions in all waves.

Indicator	1995	1996	1998	2000	2001	2002	2003
sample size	5,011	5,305	5,717	6,221	7,047	7,648	7,700
expenditures (in Rubles)	5,289	4,972	3,837	4,457	5,024	5,244	5,919
health (between 1 and 5)	3.10	3.10	3.10	3.08	3.07	3.08	3.09
Schooling (in years)	5.01	5.35	5.73	6.20	6.53	6.79	7.05

Table 1: Mean of the three indicators of well-being

Figures 2 and 3 show the evolution of the S-Gini inequality index of the three dimensions separately for $\delta = 2$ (the standard Gini coefficient) and for a more bottom sensitive member of the S-Gini class ($\delta = 5$). The inequality measures are normalized such that 1995=100.

We construct a well-being measure by expression (1). We limit ourselves to the Cobb-Douglas well-being measure ($\beta = 0$) because of the very different nature of the dimensions. Every dimension is rescaled by dividing the outcomes through the dimension-wise mean in 2000. The well-being measure is cardinalized by their equally distributed outcome. We use equal dimension weights ($w_j = 1/3$) which imply the marginal rates of substitution for an individual with average outcome vector in 2000 that are given in table

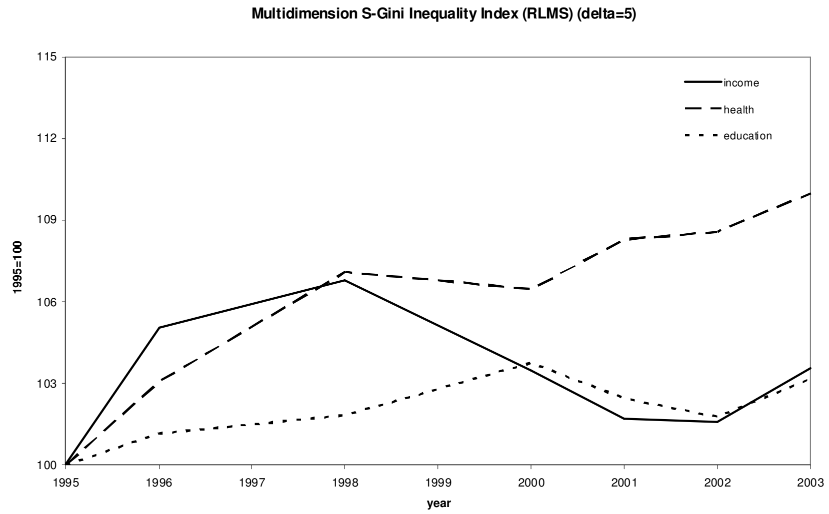

Figure 3: The evolution of Russian inequality measured by a dimension-wise S-Gini inequality index (for $\delta = 5$).

2. An individual with expenditures of 4,457 rubbles, is willing to pay 1,446 rubbles for an increase of 1 point on the 5 point health scale and 718 rubbles for one year of schooling more. Or, for instance about 300 rubbles a year for not suffering diabetes *ceteris paribus*. These figures stand to reason.

MRS	Income	Health	Schooling
Income	1		
Health	-1,446	1	
Schooling	-718	-0.50	1

Table 2: Implied Marginal Rates of Substitution between the dimensions of well-being. RLMS, 2000.

In figure 4 and 5, the evolution of multidimensional well-being inequality measured by expression (8) is given by the full black line. For reference, the grey lines depict the dimension-wise trends in inequality. The black dashed line presents the trend of well-being inequality measured by an alternative index I_{GW} that belongs to the class of normative multidimensional generalized Gini indices as characterized by Gajdos and Weymark (2005). The underlying S-Gini social evaluation function equals,

$$\underline{W}_{GW}(X|\delta,\beta,w_j) = \left[\sum_{j=1}^{m} w_j \left(\sum_{i=1}^{n}\left(\left(\frac{i}{n}\right)^{\delta} - \left(\frac{i-1}{n}\right)^{\delta}\right) x_{[i]}^{j}\right)^{\beta}\right]^{(1/\beta)}, \qquad (9)$$

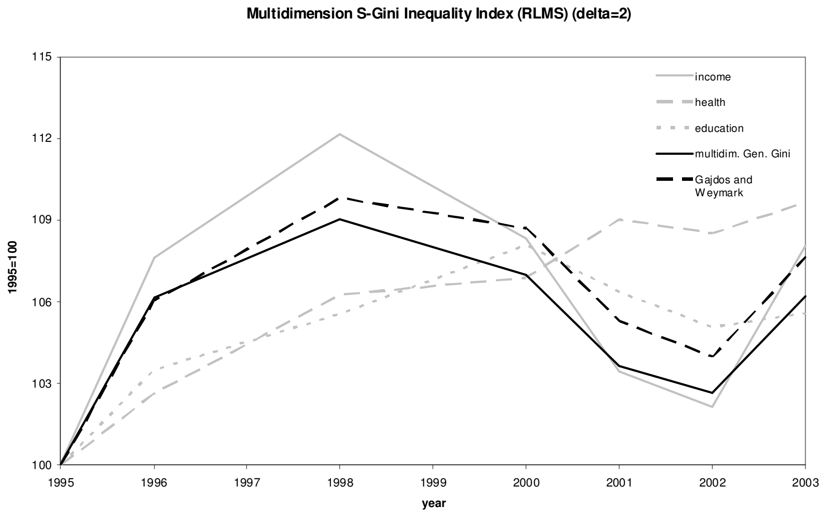

Figure 4: The evolution of Russian inequality measured by a multidimensional S-Gini inequality index (for $\delta = 2$).

where $[i]$ denotes the position of the individual in the respective dimension j. When comparing the above expression with expression (6) it is clear that both societal well-being functions use the same aggregation procedure across dimensions and individuals, but that the sequence of aggregating is switched. The social evaluation function $\underline{W}_{GW}$ aggregates first across individuals and then across dimensions.

One observes from figures 4 and 5 that for $\delta = 2$ the measure proposed in this paper I_R shows a smaller increase in inequality relative to 1995 than I_{GW}, whereas for $\delta = 5$ the opposite is the case. Over the period, correlation between the dimensions increased.[21] The higher δ, the more the index proposed in this paper increases with increasing correlation, whereas I_{GW} remains for all δ insensitive to correlation, which makes that the picture for high bottom sensitivity is less rosy for our measure than it is for I_{GW} . The selection of the most appropriate sequence of aggregation is therefore not only important from a theoretical perspective but also from an empirical one.

Finally, figure 6 depicts the normative space for $\delta > 1$ and $\beta < 1$, that is the space for which UM is satisfied. Every point in the normative space summarizes the value judge-

[21] The pair wise rank-correlation coefficients between expenditures and education; expenditures and health; and education and health increased respectively from 0.13 to 0.17, from 0.05 to 0.15 and from 0.51 to 0.66 between 1995 and 2003. See also Decancq (2008) on the measurement of correlation and dependence between the dimensions of Russian well-being.

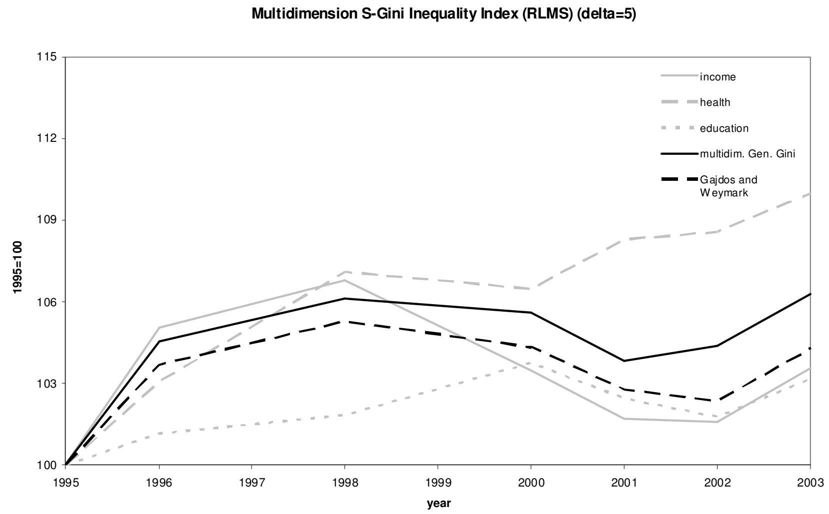

Figure 5: The evolution of Russian inequality measured by a multidimensional S-Gini inequality index (for $\delta = 5$).

ments of the external observer on the bottom sensitivity and degree of substitutability. The red area consists of the couples where inequality decreased after an unfair rearrangement in all waves, the blue area shows the ethical viewpoints for which for some waves the unfair rearrangement leads to an increase and for some waves to a decrease, and the green area presents the couples for which the unfair rearrangement leads to an increase in inequality. Taking the viewpoint that an unfair rearrangement should not lead to a decrease in inequality limits the normative space to the green points. In other words, for $\beta = 0$ we see that δ should be not smaller than 1.7.

6.2 Well-being Inequality within Indonesian ethnic groups

The second example considers the distributions of the three major ethnic groups in Indonesia in the year 2000. Two-thirds of the total population in the country belong to either the Java, Sunda, or Betawi groups (the last third of the population belongs to one of the twenty other groups normally defined).[22] Despite efforts to build a shared identity in this multi-ethnic country ('Unity in Diversity' is the national motto) by the long-lasting president Major Suharto, sectarian tensions in the last decade have lead to an increase in violent confrontations among different ethnic and religious groups. One standard explanation for the increase in conflicts relates to an deepening of inequalities

[22] Ethnic divisions in Indonesia are directly associated with geographical distribution across the Java Island. Javanese live predominantly in East and Central Java, Sundanese in West Java, while Betawis live almost exclusively in Jakarta, Indonesia's capital city at the northwest coast of the Island.

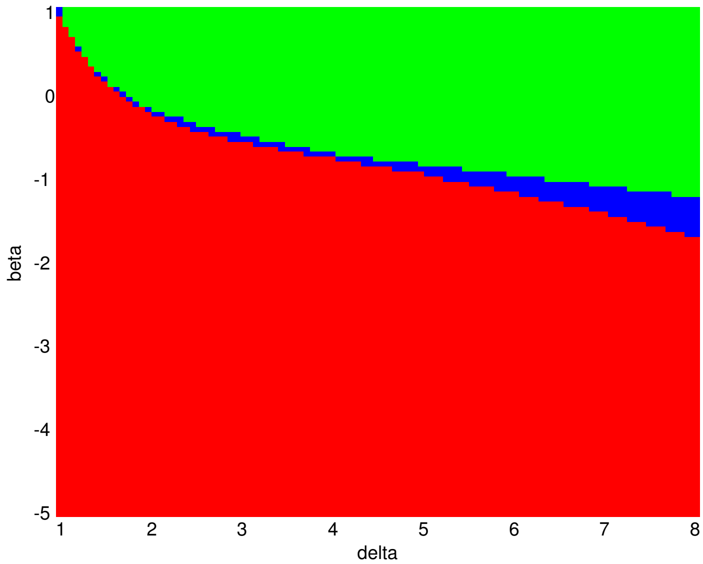

Figure 6: Compliance of the unfair rearrangement principle in the normative (δ, β)-space.

between ethnic groups in times when the country was experiencing one of the most serious economic and political crises (Betrand 2004, Brown 2005, Mancini 2005). These inequalities are not only economic ones but also in relation to the differential access to the country's resources. For instance, the Javanese is the largest and most politically dominant group, which maybe explains why being the poorest of the three groups, it has on average fairly good health levels. On the other hand, the richest group -Betawi- does not have the highest levels of education. These observed patterns call for a multidimensional approach to well-being. Although differences between ethnic groups can be important, most of the inequalities happen within groups. In fact, it is sometimes argued that the extent to which intergroup inequality can lead to conflict depends on the level of homogeneity inside groups (Zhang and Kanbur, 2003). The example used in this paper focuses on within-ethnic groups inequalities of well-being, this understood as being multidimensional.

We use data from the Indonesian Family Life Survey (IFLS), a longitudinal socioeconomic and health survey carried out by RAND-UCLA and the Demographic Institute of the University of Indonesia.[23] The IFLS was previously conducted in 1993, 1997, and 1998, but data on health status is publicly available only for 2000. In that year, approximately 10,400 households and 39,000 individuals were interviewed and for almost all of them

[23] For details on the survey, the reader is referred to Strauss *et al.* (2004)

we have complete information on the variables used. Of the total, we focus on people belonging to one of the three major ethnic groups, leaving us with almost 20.000 individuals. We use the following three indicators of well-being: equivalent real household expenditure, level of hemoglobin, and years of education of the head of the household. The square root of household size is used as equivalence scale.[24] Hemoglobin levels, expressed in grams per deciliter (gr/dl), are used as the health indicator as they indicate iron deficiency in the blood where '...[i]ron deficiency is thought to be the most common nutritional deficiency in the world today' (Thomas *et al.* 2005 p.4; see also WHO 2001 and Thomas and Frankenberg 2002). Given that normal values of hemoglobin depend on sex and age, we adjusted individual values to transform them into equivalent adult levels, using the threshold values from the WHO 2001 report (table 6, chapter 7). Individuals' ethnicity is taken from the response given by the head of the household to the following question "Which ethnic group is primarily influential in daily activities of your household?", where answers were classified into twenty-five groups, including 'Others'. Table 3 shows the average values of the three dimensions for each of the ethnic groups.

Indicator	Javanese	Sundanese	Betawis
fraction of population			
expenditure (in Rp.)	281	298	330
education (in years)	6.33	6.72	6.65
health (in g/dl)	13.96	13.85	14.01

Table 3: Mean of the three indicators of well-being

We can see from the simple averages that the best performer in one dimension is not necessarily the best in the other two. A similar picture can be observed for the dimension-wise inequality indices. The three bottom rows of figures 7 and 8 present the S-Gini inequality indices of the three dimensions separately for $\delta = 2$ (the standard Gini coefficient) and for a more bottom sensitive member of the S-Gini class ($\delta = 5$). The largest group (Java) is used as the reference point (=100). While inequality in expenditure and health seem to correspond quite closely, the ranking of distributions obtained is rather different when looking at educational inequality. In particular, while the Sundanese present the highest levels of health and economic inequality they appear to have the most equal educational distribution. And almost the opposite occurs with people belonging to the Betawi group - having the most equal distributions in expenditure and health, they perform rather poorly on educational equality. This brings the question on how to aggregate these three dimensions to the fore.

[24] Nominal per capita expenditure data are adjusted using a temporal deflator and a spatial deflator (regional poverty lines) (Strauss *et al.*, 2004).

Similarly to the previous exercise with the Russian data, we compute the well-being measure using expression (1). Once again, each dimension is rescaled by dividing the outcomes through their respective overall mean, and we assume equal weighting across dimensions. The implied marginal rates of substitution for an individual with average outcome vector are given in table 4. One unit increase in the level of hemoglobin can be compensated with a decrease of Rp. 42,095 (7.5% of average expenditure for the Javanese), while an extra year of schooling is assumed to be replaceable -in terms of well-being units- by Rp. 91,047 or with a decrease in levels of hemoglobin of 2 grams per deciliter.

MRS	Expenditure	Health	Schooling
Expenditure	1		
Health	-42,095	1	
Schooling	-91,047	-2.16	1

Table 4: Implied Marginal Rates of Substitution between the dimensions of well-being. Indonesia, 2000.

The resulting multidimensional inequality indices for each of the three ethnic groups are represented in figures 7 and 8. The first row presents I_{GW} and the second row I_R as it is defined in expression (8) of this paper. For $\delta = 2$, the ordering of distributions across ethnic groups differs whether one concentrates on our index or on I_{GW} . While both indices agree that the Javanese have the most unequal distribution of well-being, our index will rank the Sundanese as more equal than Betawis while the opposite is true for the other index.

Finally, figure 9 depicts the normative (δ, β) space for which UM is satisfied. The red area consists of the points where inequality is lower after an unfair rearrangement in all waves and the blue area collects the ethical viewpoints for which for some waves the unfair rearrangement leads to an increase and for some waves to a decrease. The green area presents the couples for which the unfair rearrangement leads to an increase in inequality, this is the preferred area according to Dardanoni's principle (1995). In other words, for $\beta = 0$ we see that δ should be not smaller than 2.8, so that the multidimensional standard Gini ($\delta = 2$), *de*creases after an unfair rearrangement.

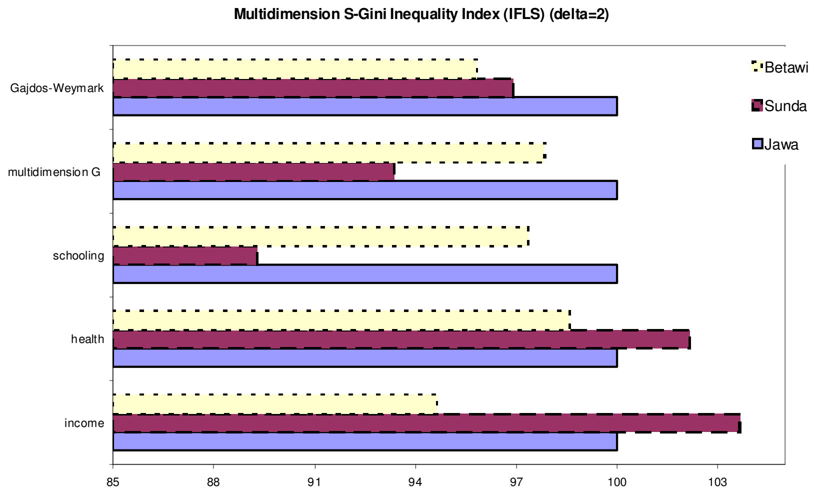

Figure 7: Indonesian inequality measured by an S-Gini inequality index (for $\delta = 2$).

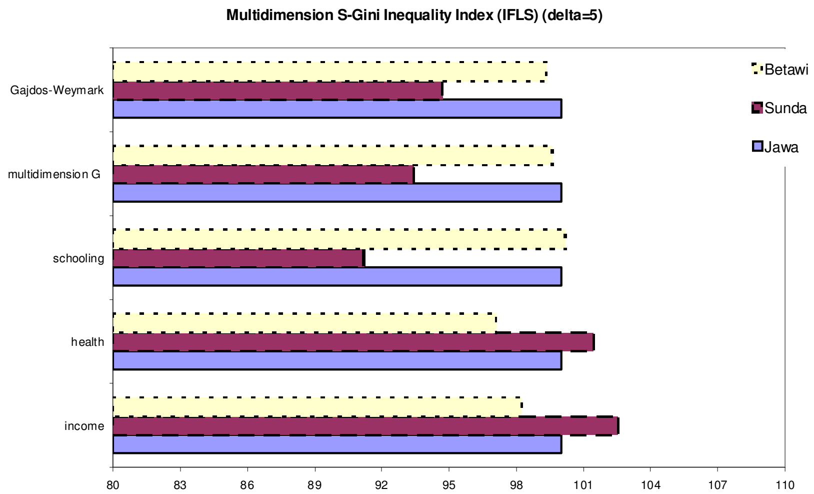

Figure 8: Indonesian inequality measured by an S-Gini inequality index (for $\delta = 5$).

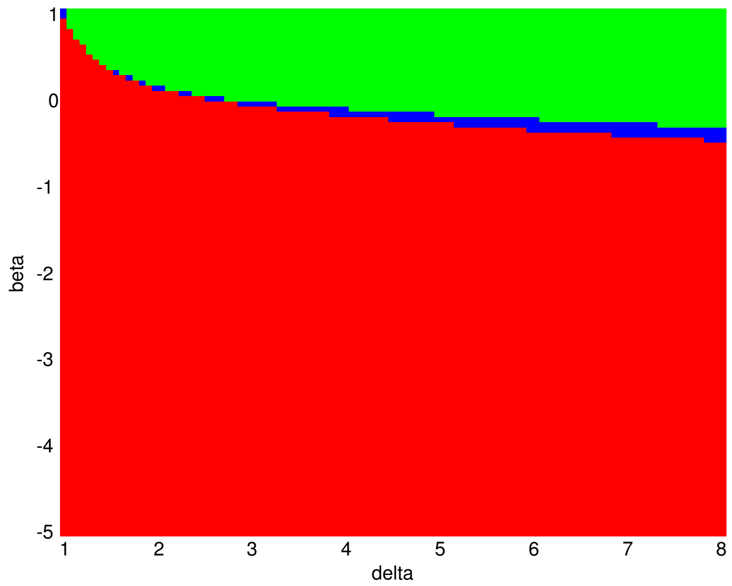

Figure 9: Compliance of the unfair rearrangement principle in the normative (δ, β)-space.

6.3 World well-being inequality

As a final example, we look at the evolution of well-being inequality between the countries of the world. By now, the evolution of *income* inequality is relatively well-documented in the literature.[25] Much less is known on the evolution *well-being* inequality taking also other dimensions such as health and education into account.

In their paper Decancq, Decoster and Schokkaert (2008) investigate the trend of well-being inequality between 1975 and 2000 based on aggregated data for 97 countries for four indicators of well-being: GDP *per capita*, life expectancy, literacy and school enrolment rate.[26] We make use of the same dataset here, but use a distinct underlying multidimensional social evaluation function. Whereas the underlying compound social evaluation function proposed in this paper is given by expression (6), Decancq *et al.* make use of the following social evaluation function:

$$\underline{W}_{DDS}(Z|\beta, w_j, \varepsilon) = \frac{1}{1-\varepsilon} \sum_{i=1}^{n} \left(\sum_{j=1}^{m} w_j \left(z_i^j \right)^{\beta} \right)^{1-\varepsilon/\beta} . \tag{10}$$

When comparing expression (10) with the social evaluation function proposed in this

[25] Milanovic (2005) or Anand and Segal (2008) give a recent overview of the literature and the impact of alternative choices on the construction of the data, population weighting and inequality aversion on the obtained results.

[26] For a more detailed treatment of the data, see Decancq, Decoster and Schokkaert (2008).

paper (expression 6), one notices that though the aggregation across dimensions uses the same functional form, the aggregation across individuals is very different. The approach of the present paper uses a Gini-type social evaluation function that is separable on the domain of the ordered well-being functions, whereas expression (10) uses a strict separable social evaluation function of the Atkinson-type in which the individual contribution to societal well-being is independent of the rank of the individual in the total well-being distribution. We compare the results of both procedures with the results obtained by the procedure proposed by Gajdos and Weymark (2005), which first aggregates across individuals and then across dimensions.

To stay as close as close as possible to the setting of Decancq *et al.* (2008), we do not use population weights for the different countries. In other words, we investigate the inequality in the UN-assembly, treating the outcome vectors of highly-populated countries in the same way as the vectors of unpopulated ones. The four indicators of well-being are standardized by,

$$z_i^j = \frac{x_i^j - x_{\min}^j}{x_{\max}^j - x_{\min}^j} \text{ for all } j \text{ in } \mathcal{M}. \tag{11}$$

with the dimension-wise minimal and maximal values ($x_{\min}^j$ and $x_{\max}^j$) summarized in table 5. The last column of the table contains the dimension weights which are equal to the ones used in the Human Development Index (HDI). For $\beta = 1$, the well-being function $S(z_i)$ is very similar to the HDI.[27]

Indicator	$x_{\min}^j$	$x_{\max}^j$	w_j
GDP per capita	100	40000	0.333
Longevity	25	85	0.333
Literacy rate	0	100	0.222
Enrolment rate	0	100	0.111

Table 5: Goalposts and weights in the Human Development Index.

Figure 10 and 11 summarize the dimension-wise S-Gini inequality indices of the four indicators after transformation, for two different bottom sensitivity parameters $\delta = 2$ and $\delta = 5$. Over the period income inequality increases, educational inequality drops and health inequality shows a distinct U-shaped pattern. The more weight is given to the bottom of the distribution, the higher the increase in inequality in longevity is after

[27]Note however, that the HDI uses logarithm of GDP *per capita* as dimension, whereas here we use GDP *per capita* as such.

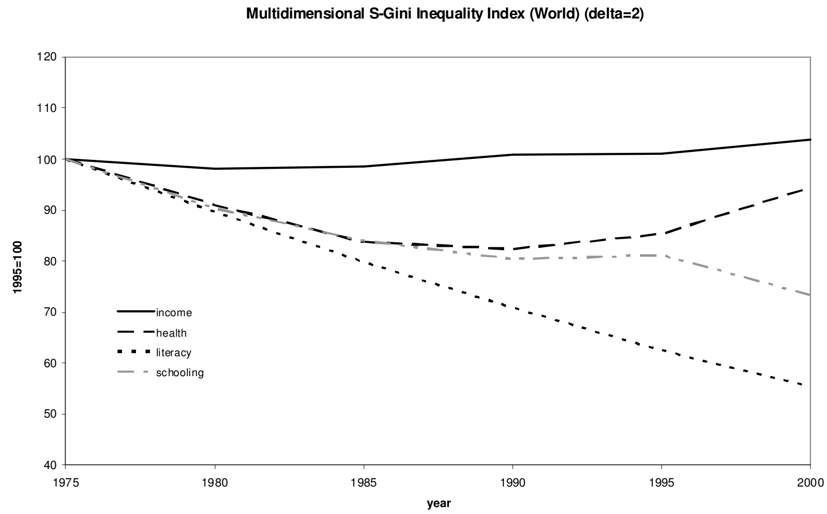

Figure 10: The evolution of inequality within each dimension of world well-being, measured by a S-Gini index (for $\delta = 2$).

1985-1990. In the literature this increase in inequality has been mainly attributed to the HIV-epidemic (Becker, Philipson and Soares 2005).

Figure 12 shows the evolution of well-being inequality measured by the multidimensional S-Gini index I_R derived in this paper (expression 8) for alternative degrees of substitutability β and δ equal to 2 and 5, respectively. Figure 13 depicts the evolution according to the I_{GW} the multidimensional S-Gini index (a special case of the measure proposed in Gajdos and Weymark 2005). Finally, Figure 14 replicates the results of Decancq *et al.* (2008) based on the underlying social evaluation function in expression (10). The general picture is one of a U-shaped pattern. Both measures based on a rank-dependent aggregation (figure 12 and 13) shows a pattern similar, whereas the measures based on (10) shows a steeper increase after 1985. Of course the results cannot be compared by the completely different role played by δ and ε, nevertheless the choice of family of aggregation procedure across individuals is decisive for the obtained results and should be considered with care .

Finally, figure 15 can be interpreted similarly as figure 6.[28] The red area shows the value judgements captured by β and δ for which the measure never increases after an unfair rearrangement, the points where for some waves the measure increases and for others decreases are blue, and the green area are the points for which the measure increases

[28] Note that a similar figure for I_{GW} is completely red given its insensitivity to correlation in general, whereas for I_{DDS} (in the $\beta - \varepsilon$ normative space) it is red below the line $\varepsilon + \beta = 1$ and green above.

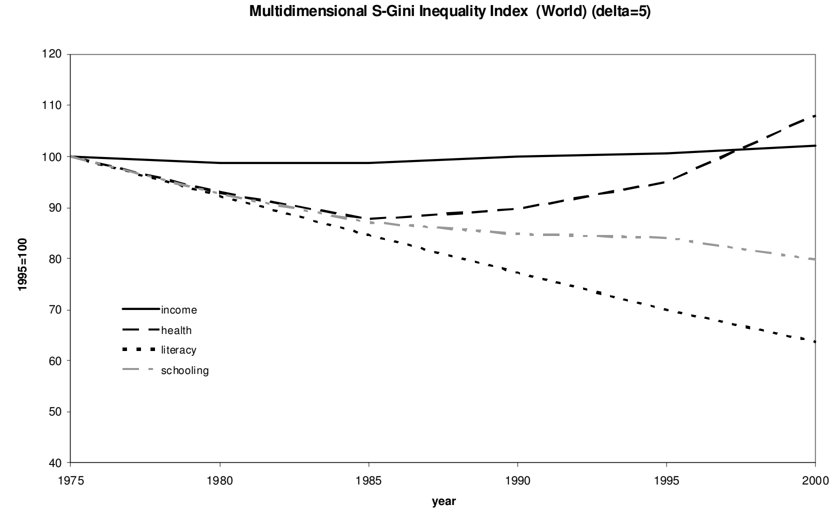

Figure 11: The evolution of inequality within each dimension of world well-being, measured by a S-Gini index (for $\delta = 5$).

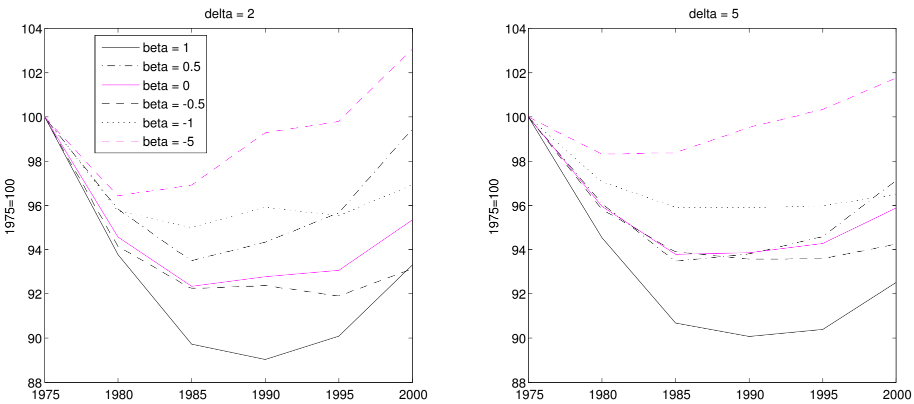

Figure 12: The evolution of world well-being inequality measured by the multidimensional S-Gini inequality index proposed in this paper for different β and δ equal to 2 and 5.

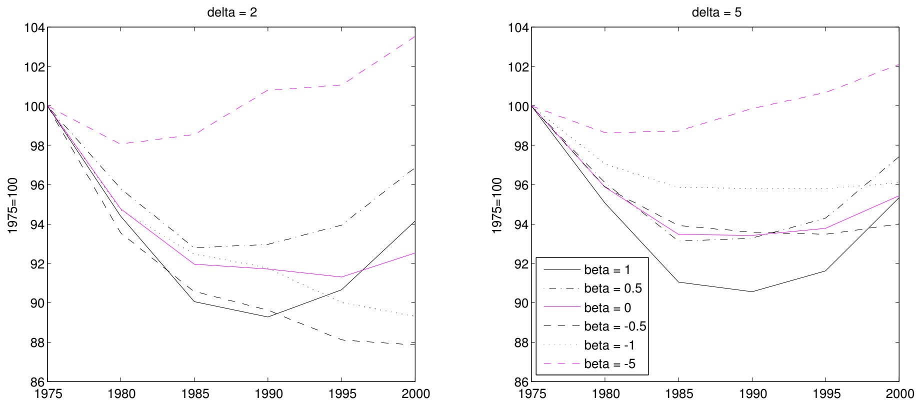

Figure 13: The evolution of world well-being inequality measured by the multidimensional S-Gini inequality index, a member of the class characterized by Gajdos and Weymark (2005) for different β and δ equal to 2 and 5.

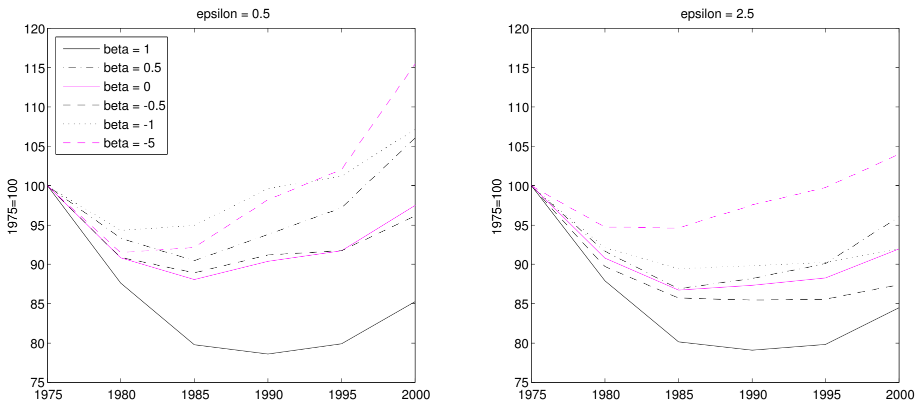

Figure 14: The evolution of world well-being inequality measured by the multidimensional Atkinson inequality index used by Decancq, Decoster and Schokkaert (2008) for different β and ε equal to 0.5 and 2.5 respectively.

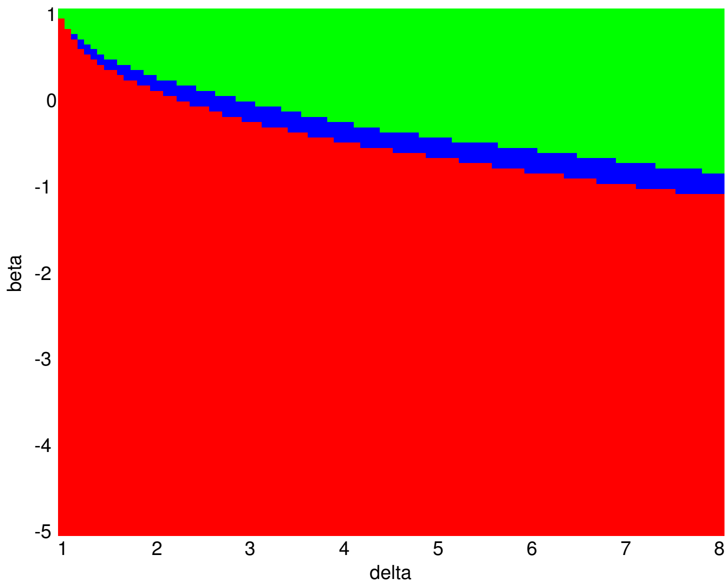

Figure 15: Compliance of the unfair rearrangement principle in the normative (δ, β)-space.

after an unfair rearrangement. For the used world dataset, the HDI-weights and a degree of substitutability $\beta = 0$, the minimal δ for compliance with the unfair rearrangement principle is about 2.8, which is considerably higher than the one found in the Russian dataset and very similar to the value obtained for the Indonesian dataset. Again, the standard multidimensional Gini ($\delta = 2$) lies outside the green area, so that for this case an unfair rearrangement would lead to a decrease in inequality.

7 Conclusion

In this paper we used an intuitive two step procedure to characterize a multidimensional single parameter Gini social evaluation function. The aggregation across dimensions uses a CES or Cobb-Douglas well-being function. Across individuals, we obtain a rank-dependent aggregation procedure, where the welfare weights depend on the ranks of the individual in the well-being distribution which allows individuals' well-being to be evaluated in comparisons to others in the society.

Conditions for the compliance of the obtained single parameter Gini social evaluation function to a multidimensional Pigou-Dalton transfer-principle as well as to correlation increasing majorization are derived. The latter condition shows that researchers willing to work in a two-step multidimensional rank-dependent framework, may be forced to use a highly bottom sensitive aggregation procedure across individuals.

The obtained single parameter Gini social evaluation function allows us to derive a relative multidimensional measure of inequality. This measure has been applied to household data from Russia and Indonesia and to aggregated international data on three dimensions of well-being: material standard of living, health and education. Our results shed some light on several important normative choices that matter empirically, especially the sequence of the aggregation procedures across dimensions and individuals; and the choice of the aggregation procedure across individuals.

A further extension within the two-step rank-dependent framework presented here, includes the study of a translation invariant social evaluation function and absolute inequality measures. Furthermore, the characterization of a one-step multidimensional generalized Gini social evaluation function that is sensitive to the correlation between the dimensions remains an intriguing theoretical puzzle for further research.

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Appendix A

Proposition 4. *A compound social evaluation relation* $\underline{\mathsf{R}}_{\succeq}$ *satisfying the properties specified in Propositions 1b and 3 respectively, also satisfies UM if the representation of* $\underline{\mathsf{R}}_{\succeq}$ *satisfies* $\beta < 1$ and $\delta > 1$.

Proof. Let $Y = BX$ with B an $n \times n$ bistochastic matrix. According to Kolm (1977) $[S(x_{[1]}), S(x_{[2]}), ..., S(x_{[n]})]'$ is strictly generalized Lorenz dominated by $[S(y_{[1]}), S(y_{[2]}), ..., S(y_{[n]})]'$ if S is increasing and strictly concave (see also Weymark, 2006). If W is increasing and strictly concave $W[S(x_{[1]}), S(x_{[2]}), ..., S(x_{[n]})]' \leq W[S(x_{[1]}), S(x_{[2]}), ..., S(x_{[n]})]'$ (Lambert 2001; theorem 3.2).

The well-being function S is increasing by virtue of $\text{MON}_{\succeq}$ and strictly concave if $\beta < 1$ (Ebert, 1988; proposition 6). The social evaluation function W is increasing by virtue of $\text{MON}_{\underline{\mathsf{R}}}$ and strictly concave by $\delta > 1$ (Ebert, 1988; proposition 10). □

Proposition 5. *A compound social evaluation relation* $\underline{\mathsf{R}}_{\succeq}$ *satisfying the properties specified in Proposition 1b and 3 cannot satisfy CIM.*

Proof. Consider the distribution matrices Y and X, where Y is obtained from X after one correlation increasing transfer between individual l and k as defined in definition 1.

From the definition of a correlation increasing transfer and the monotonicity of the well-being relation ($\text{MON}_{\succeq}$), it follows that

$$S(y_k) \geq S(y_l),$$

$$\begin{aligned} S(x_l) &\geq S(y_l), S(x_k) \geq S(y_l) \text{ and} \\ S(y_k) &\geq S(x_k), S(y_k) \geq S(x_l) \end{aligned}$$

so that,

$$S(y_k) \geq S(x_l), S(x_k) \geq S(y_l). \tag{12}$$

The compound social evaluation relation $\underline{\mathsf{R}}_{\succeq}$ satisfies CIM if and only if $X \ \underline{\mathsf{R}}_{\succeq} \ Y$. By increasing the bottom sensitivity (the δ parameter) one can give more weight to the decrease of well-being of the worse-off individual l, than to increase of the better-off individual k by the correlation increasing transfer, so that for a given X, it holds that $X \ \underline{\mathsf{R}}_{\succeq} \ Y$, provided δ is "large enough".

However, to make sure that $X \ \underline{\mathsf{R}}_{\succeq} \ Y$ holds for all possible X, one has to go to extreme bottom sensitivity ($\delta = \infty$) which is excluded by MON_{R}. □

Appendix B

	health	
diabetes	-0.207***	(0.0133)
hart attack	-0.303***	(0.0159)
anemia	-0.168***	(0.0138)
smokes	-0.051***	(0.0062)
exercise	0.113***	(0.0084)
health problem	-0.451***	(0.0053)
hospitalized	-0.229***	(0.0119)
check up	-0.057***	(0.0067)
operation	-0.058***	(0.0137)
jogged	0.064***	(0.0131)
age 20	-0.070***	(0.0090)
age 30	-0.241***	(0.0093)
age 40	-0.401***	(0.0091)
age 50	-0.532***	(0.0100)
age 60	-0.694***	(0.0101)
age 70	-0.932***	(0.0116)
age 80	-1.180***	(0.0186)
age 90	-1.131***	(0.0471)
male	0.142***	(0.0057)
cons	3.676***	(0.0078)
N	58166	

Standard errors in parentheses

$* : p < 0.05, ** : p < 0.01, *** : p < 0.001$

Table 6: Health equation, RLMS, pooled.

Table 7: **Summary Statistics. Russia from 1995 to 2003**

Variables	**mean**	**sd**	**min**	**max**
	1995 (N = 5,011)			
Expenditures (in Rubles)	5289.29	5923.46	9.79	160249.9
Schooling (in years)	5.01	3.72	1.00	16.00
Health (between 1 and 5)	3.10	0.44	1.45	4.00
	1996 (N = 5,305)			
Expenditures (in Rubles)	4972.10	6276.54	60.98	177450.4
Schooling (in years)	5.35	4.02	1.00	16.00
Health (between 1 and 5)	3.10	0.45	1.53	4.00
	1998 (N = 5,717)			
Expenditures (in Rubles)	3837.10	6991.65	35.46	203582.5
Schooling (in years)	5.73	4.29	1.00	16.00
Health (between 1 and 5)	3.10	0.47	1.39	4.00
	2000 (N = 6,221)			
Expenditures (in Rubles)	4457.35	6276.64	30.37	124255.7
Schooling (in years)	6.20	4.60	1.00	16.00
Health (between 1 and 5)	3.08	0.46	1.39	4.00
	2001 (N = 7,047)			
Expenditures (in Rubles)	5023.62	6600.78	60.52	251334.3
Schooling (in years)	6.53	4.72	1.00	16.00
Health (between 1 and 5)	3.07	0.47	1.49	4.00
	2002 (N = 7,648)			
Expenditures (in Rubles)	5244.46	6228.05	10.39	181401.2
Schooling (in years)	6.79	4.81	1.00	16.00
Health (between 1 and 5)	3.08	0.47	1.38	4.00
	2003 (N = 7,700)			
Expenditures (in Rubles)	5919.61	9452.21	61.15	235387.2
Schooling (in years)	7.05	4.94	1.00	16.00
Health (between 1 and 5)	3.09	0.48	1.47	4.00

Source: RLMS, authors' calculations

Table 8: **Summary Statistics. Indoensia 2000, by ethnic groups**

Variables	**mean**	**sd**	**min**	**max**
	Jawa (N = 13,562)			
Expenditures (in Rp.)	280,571	303,719	20,348	5,236,150
Schooling (in years)	6.33	4.57	0.01	19.00
Health (in g/dl)	13.96	1.71	3.60	25.82
	Sunda (N = 4,257)			
Expenditures (in Rp.)	297,653	336,616	24,391	6,066,339
Schooling (in years)	6.72	4.38	0.01	19.00
Health (in g/dl)	13.85	1.72	3.50	19.40
	Betawi (N = 1,791)			
Expenditures (in Rp.)	329,907	342,960	42,577	3,901,813
Schooling (in years)	6.65	4.61	0.01	17.00
Health (in g/dl)	14.01	1.69	3.15	20.12

Source: IFLS 2000, authors' calculations

Table 9: **Summary Statistics. World distribution from 1975 to 2000**

Variables	mean	sd	min	max
	1975 (N = 97)			
GDP per capita	2354.85	2263.47	190.00	8670.00
Life expectancy (in years)	60.24	10.95	39.10	75.50
Literacy rate	65.67	30.09	6.72	99.80
Secondary enrolment	42.95	30.04	1.27	108.00
	1980 (N = 97)			
GDP per capita	3818.97	3630.80	330.00	13500.00
Life expectancy (in years)	62.21	10.55	40.30	76.60
Literacy rate	68.81	28.46	7.95	99.80
Secondary enrolment	48.29	30.35	2.70	109.00
	1985 (N = 97)			
GDP per capita	5164.95	4978.01	380.00	18000.00
Life expectancy (in years)	64.03	10.26	41.00	77.70
Literacy rate	71.88	26.74	9.57	99.80
Secondary enrolment	53.34	31.11	3.57	117.00
	1990 (N = 97)			
GDP per capita	6757.22	6746.27	420.00	25200.00
Life expectancy (in years)	65.25	10.55	40.20	78.80
Literacy rate	74.78	25.02	11.40	99.80
Secondary enrolment	55.90	31.17	5.62	120.00
	1995 (N = 97)			
GDP per capita	8194.74	8196.63	520.00	32800.00
Life expectancy (in years)	65.95	11.29	38.20	79.50
Literacy rate	77.55	23.26	13.50	99.80
Secondary enrolment	63.52	35.99	6.60	146.00
	2000 (N = 97)			
GDP per capita	10344.54	10914.53	600.00	56300.00
Life expectancy (in years)	66.16	12.80	38.00	81.10
Literacy rate	79.94	21.67	16.00	99.70
Secondary enrolment	70.41	36.31	6.46	161.00

Source: Human Development Report 2005, authors' calculations